# Extension-ranking Semantics for Abstract Argumentation

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In this paper, we present a general framework for ranking sets of arguments in abstract argumentation frameworks based on their plausibility of acceptance. We present a generalisation of Dung's extension semantics as *extension-ranking semantics*, which induce a preorder over the power set of all arguments, allowing us to state that one set is "closer" to being acceptable than another. To evaluate the extension-ranking semantics, we introduce a number of principles that a well-behaved extension-ranking semantics should satisfy. We consider several simple base relations, each of which models a single central aspect of argumentative reasoning. The combination of these base relations provides us with a family of extension-ranking semantics.

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## 1 Introduction

Formal argumentation (Atkinson et al. 2017) is concerned with models of rational decision-making based on representations of arguments and their relations. A particularly important approach is that of *abstract argumentation frameworks* (AF) (Dung 1995), which represent argumentative scenarios as directed graphs. Here, *arguments* are identified by vertices, and an *attack* from one argument to another is represented as a directed edge. Reasoning is usually performed in abstract argumentation by considering *extensions*, i.e., sets of arguments that are jointly acceptable, given some formal account of "acceptability". One way to determine the acceptability of a set is to look at the internal conflicts of this set. A set of arguments is considered "acceptable" if it is internally consistent, so that no two arguments within a set attack each other. Another concept to establish the acceptability of a set of arguments is *admissibility*, which states that a set is only acceptable if it can defend itself against any threat. Acceptability concepts such as admissibility give rise to *extension semantics*. These semantics distinguish between "acceptable" sets of arguments and "non-acceptable" sets of arguments.

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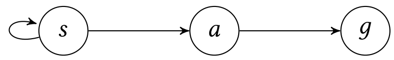


Fig. 1. AF $F_1$ from Example 1.

The binary classification inherent in extension semantics proves overly restrictive, particularly when the goal is to compare sets of arguments based on their acceptability. This limitation becomes evident when examining more nuanced scenarios, such as the law example detailed below (for the formal definitions, see Section 2).

Example 1. *John is the prime suspect in a murder case and possesses an alibi for the time of the crime. A witness, Jack, claims having seen John at the crime scene. However, it is later revealed that Jack was intoxicated, rendering his testimony unreliable and subject to scrutiny. This scenario can be modelled using an abstract argumentation framework* $F_1 = (\{g, a, s\}, \{(a, g), (s, a), (s, s)\})$ *where:*

- $g$ *represents the argument* "John is guilty",
- $a$ *represents the argument* "John has an alibi",
- $s$ *represents the argument* "John was seen at the scene of the crime".

*In this framework, the argument* $g$ *is attacked by* $a$*, and* $a$ *is attacked by* $s$*. Notably,* $s$ *is unreliable and therefore attacks itself. The corresponding AF is depicted in Figure 1. The judge decides which position is most plausible. John is either* guilty *or not, implying that* $g$ *is either part of an acceptable set or not. When comparing the two positions of* $\{a\}$ *and* $\{g, s\}$*, the judge concludes that neither set is acceptable with respect to admissibility. This is because neither set can defend itself against its attackers. However, while the set* $\{a\}$ *does not defend itself against the attack of* $s$*, at least this set has no internal conflicts. In contrast, the set* $\{g, s\}$ *has an internal conflict due to the self-attacking nature of* $s$*.*

In Example 1 above, we observe an order between two non-acceptable sets. The set $\{a\}$ is "closer" to being acceptable than the set $\{g, s\}$. Extending this analysis, we examine the relationship with the set $\{a, g, s\}$. Both $\{g, s\}$ and $\{a, g, s\}$ share one conflict (the self-attack of $s$), but $\{a, g, s\}$ has additional conflicts. Consequently, $\{g, s\}$ is considered "closer" to being acceptable than $\{a, g, s\}$, and similarly, the set $\{a\}$ is also "closer" to being acceptable than $\{a, g, s\}$. This establishes an order for these sets based on their plausibility of acceptance. The above example highlights the need for a more fine-grained evaluation beyond the binary classification of extension semantics.

In the literature (Amgoud and Ben-Naim 2013; Amgoud, Ben-Naim, et al. 2016; Bonzon et al. 2016b; Grossi and Modgil 2019; Yun et al. 2018), we can find a number of approaches to determine the relative degree to plausibly accept an individual argument. These approaches, known as *argument-ranking*, *graded* or *gradual* semantics, associate each abstract argumentation framework with an ordering of arguments according to their relative degrees of justification, strength or plausibility. However, the plausibility of acceptance of each argument cannot be directly used to infer the plausibility of acceptance of a set of arguments, because these approaches do not facilitate the representation of a set of arguments as "jointly" acceptable. Specifically, two arguments with a high degree of plausibility to be accepted may not be accepted simultaneously if they are in conflict with each other.

In this paper, we introduce the notion of *extension-ranking semantics* to provide a more fine-grained evaluation than binary classification, by ranking sets of arguments according to their acceptance. Such rankings are useful for decision-making under constraints, where we need solutions (typically represented by extensions of an argumentation framework) that satisfy constraints that may not be satisfiable by the set of arguments under an extension semantics. In such a situation, we can select the most plausible set of arguments among those that satisfy the constraints. Another application is belief dynamics (Booth, Kaci, et al. 2013; Coste-Marquis et al. 2014; Diller et al. 2018), where rankings over sets of arguments can be used as a relation of epistemic entrenchment. Moreover, unlike argument-ranking semantics, we properly generalise extension semantics, in the sense that the set of most plausible sets is the set of extensions under a traditional semantics.

We contribute to the research question of ranking sets of arguments by generalising extension semantics, which allows us to state whether one set is, e. g., "more admissible" than another. More precisely, the main contributions of this paper are as follows.

- We present *extension-ranking semantics* $\tau$, which maps an argumentation framework $AF$ to a preorder $\sqsupseteq^{\tau}_{AF}$ over all sets of arguments. Intuitively, for two sets $E$ and $E'$ of $AF$ with $E \sqsupseteq^{\tau}_{AF} E'$ we say that $E$ is at least as plausible to be accepted as $E'$ wrt. $\tau$. We will show later that any extension semantics can be defined in terms of an extension-ranking semantics, however, the general framework of extension-ranking semantics is more expressive than that.
- We define a set of principles for evaluating and developing new approaches to extension-ranking semantics. These principles are all modelling aspects that an intuitively well-behaved extension-ranking semantics should satisfy.
- We develop very general approaches to rank sets of arguments by splitting central aspects of argumentative reasoning into conceptually simple base relations and aggregating these relations to establish the plausibility of acceptance of each set, in turn defining generalisations of the classical Dung *extension semantics*.

The remainder of this paper is structured as follows. In Section 2 necessary background information about abstract argumentation is recalled. Extension-ranking semantics are presented and put into context with extension semantics in Section 3. Section 4 introduces a set of principles for evaluating and developing extension-ranking semantics. Each principle addresses a specific aspect that a well-behaved extension-ranking semantics should satisfy. In Section 5 argumentative reasoning is broken down into conceptually simple base relations. Each base relation focuses on a single aspect of argumentation, enabling the comparison of two sets of arguments according to that specific relation. Section 6 combines the base relations to define families of extension-ranking semantics. In Finally, Section 7 summarises the results of this work, discusses related work not covered in previous sections, and outlines future research directions.

This paper is a significantly extended version of a paper published in the proceedings of the Thirtieth International Joint Conference on Artificial Intelligence, IJCAI 2021 (Skiba et al. 2021). This version contains, in addition to the proofs, additional extension-ranking semantics and their analysis. We streamlined some definitions, for example the most plausible sets are now the maximal sets instead of the minimal sets (see Definition 8). Compared to (Skiba et al. 2021), we present a general framework to define extension-ranking semantics by defining first base relations and then propose two combinations approaches for these base relations. We extended the set of base relations by cardinality-based ones (see Section 5.2). For the combination approaches, we further discussed the lexicographic combination and introduce a Copeland-based combination (Section 6.2). Also we extended the discussion of the principles by investigating the compatibility among them.

## 2 Preliminaries

*Abstract argumentation framework* (Dung 1995) is a formalism for reasoning with conflicting pieces of information, such as evidence in a criminal case or arguments in a discussion about holiday plans, using arguments and attacks between them. Let $\mathcal{U}$ be the infinte background set.

Definition 1. *An* abstract argumentation framework *(AF) is a directed graph $F$, where $A \subseteq \mathcal{U}$ is a finite set of* arguments *and $R$ is an* attack relation *$R \subseteq A \times A$.*

We denote with $\mathcal{AF}$ the set of all abstract argumentation frameworks, which can be constructed from $\mathcal{U}$. For an abstract argumentation framework $F = (A, R) \in \mathcal{AF}$, an argument $a \in A$ is said to *attack* an argument $b \in A$ if $(a, b) \in R$. A set $E \subseteq A$ is said to *defend* an argument $a \in A$ if every argument $b \in A$ that attacks $a$ is, in turn, attacked by some argument $c \in E$. For $a \in A$, we define:

$$a^-_F = \{b \mid (b, a) \in R\} \quad \text{and} \quad a^+_F = \{b \mid (a, b) \in R\}.$$

In other words, $a_F^-$ is the set of attackers of $a$ and $a_F^+$ is the set of arguments attacked by $a$. For a set of arguments $E \subseteq A$, we extend these definitions to $E_F^-$ and $E_F^+$ via:

$$E_F^- = \bigcup_{a \in E} a_F^- \quad \text{and} \quad E_F^+ = \bigcup_{a \in E} a_F^+.$$

If the abstract argumentation framework is clear from the context, we omit the index. Additionally, with a slight abuse of notation, we denote an attack from an argument on a set of arguments as follows: Let $F = (A, R) \in \mathcal{AF}$ be an AF, with $a \in A$, and $E, E' \subseteq A$. We denote with $(a, E) \in R$ that there exists an argument $b \in E$ such that $(a, b) \in R$. Extending this notation to $(E, E')$, it means that there exist arguments $b \in E$ and $c \in E'$ such that $(b, c) \in R$.

For two AFs $F = (A, R) \in \mathcal{AF}$ and $F' = (A', R') \in \mathcal{AF}$, we define their union by

$$F \cup F' = (A, R) \cup (A', R') = (A \cup A', R \cup R').$$

An *isomorphism* $\gamma$ between two AFs $F = (A, R) \in \mathcal{AF}$ and $F' = (A', R') \in \mathcal{AF}$ is a bijective function $\gamma : AF \to AF$ such that $(a, b) \in R$ if and only if $(\gamma(a), \gamma(b)) \in R'$ for all $a, b \in A$ and all attacks $r \in R$.

A number of semantic notions for reasoning with abstract argumentation frameworks can be found in the literature (for overviews see (Baroni, Gabbay, et al. 2018; Gabbay et al. 2021)). An common approach is the use of *extension semantics*, where sets of arguments are considered jointly acceptable. To define extension semantics, we rely on the two fundamental concepts of *conflict-freeness* and *admissibility*.

Definition 2. *Given AF $F = (A, R) \in \mathcal{AF}$, a set $E \subseteq A$ is*

- conflict-free *if and only if for all pairs $a, b \in E$, we have $(a, b) \notin R$;*
- admissible *if and only if $E$ is conflict-free and $E$ defends all its elements, i.e., $E_F^- \subseteq E_F^+$.*

We denote the sets of conflict-free and admissible sets of an abstract argumentation framework $F = (A, R)$ by $\mathsf{cf}(F)$ and $\mathsf{ad}(F)$, respectively. A property of admissibility is formalised with the *Fundamental Lemma* of Dung 1995.

Lemma 1 ((Dung 1995)). *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E \in \mathsf{ad}(F)$. If $a \in A$ is defended by $E$, then $E \cup \{a\} \in \mathsf{ad}(F)$.*

Intuitively, an argument defended by an admissible set can be added to that admissible set without violating the admissibility of the set.

Next, we define the *characteristic function* for abstract argumentation frameworks.

Definition 3. *For AF $F = (A, R) \in \mathcal{AF}$, the* characteristic function $\mathfrak{F}_F : \mathcal{P}(A) \to \mathcal{P}(A)$ *is defined for $E \subseteq A$ via:*

$$\mathfrak{F}_F(E) = \{a \in A \mid E \text{ defends } a\}.$$

The characteristic function, for a set of arguments $E$, identifies the arguments defended by $E$. We introduce the *extension semantics* proposed by Dung 1995.

Definition 4. *Let $F = (A, R) \in \mathcal{AF}$ be an AF. An admissible set $E \subseteq A$ is*

- *a* complete *extension* ($\mathsf{co}$) *if and only if $E = \mathfrak{F}_F(E)$;*
- *a* grounded *extension* ($\mathsf{gr}$) *if and only if it is the $\subseteq$-minimal complete extension;*
- *a* preferred *extension* ($\mathsf{pr}$) *if and only if it is a $\subseteq$-maximal admissible set;*
- *a* stable *extension* ($\mathsf{st}$) *if and only if $E_F^+ = A \setminus E$.*

Note that the grounded extension is unique for each abstract argumentation framework (Baroni, Gabbay, et al. 2018). Furthermore, for each abstract argumentation framework, there is always at least one extension for the

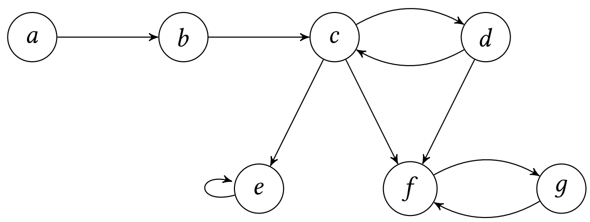


Fig. 2. AF $F_2$ from Example 3.

complete, preferred and grounded extension semantics. However, this is not necessarily the case for the stable extension semantics.

Example 2. *Consider the AF $F_1$ from Example 1. In this AF, none of the arguments are part of a stable extension. This is because each argument either cannot be defended against its attackers (arguments $a$ and $g$) or attacks itself (argument $s$). Consequently, $F_1$ has no conflict-free set which attacks every argument not contained in the set, and therefore, there is no stable extension.*

Due to of the nonexistence of stable extensions, M. W. A. Caminada et al. 2012 proposed a novel variation of the stable extension semantics, the so-called *semi-stable* extension semantics.

Definition 5. *For AF $F = (A, R) \in \mathcal{AF}$, a set $E \subseteq A$ is a* semi-stable *extension if and only if $E$ is a complete extension where $E \cup E_F^+$ is maximal with respect to set inclusion.*

The set of semi-stable extensions coincides with the set of stable extensions when the latter is not empty (M. W. A. Caminada et al. 2012).

Example 3. *Consider the abstract argumentation framework $F_2$ depicted as a directed graph in Figure 2. $F_2$ has four complete extensions:*

$$E_1 = \{a\}, \qquad E_3 = \{a, c, g\},$$
$$E_2 = \{a, g\}, \quad E_4 = \{a, d, g\}.$$

*$\{a\}$ is the grounded extension, while $\{a, c, g\}$ and $\{a, d, g\}$ are both preferred extensions, but only $\{a, c, g\}$ is a stable extension and therefore also a semi-stable extension.*

The sets of extensions of an abstract argumentation framework $F$, for the five semantics mentioned, are represented by $\mathrm{co}(F)$, $\mathrm{gr}(F)$, $\mathrm{pr}(F)$, $\mathrm{st}(F)$, and $\mathrm{sst}(F)$. These extension semantics are commonly referred to as Dung's extension semantics.

Using the extension semantics, we can determine whether each argument is accepted or not. If an argument $a$ is part of at least one $\sigma$-extension of an abstract argumentation framework $F$, then $a$ is *credulously accepted* with respect to the extension semantics $\sigma$. If $a$ is part of every $\sigma$-extension of $F$, then $a$ is *skeptically accepted* with respect to the extension semantics $\sigma$. An argument $a$ that is not part of any $\sigma$-extension is *rejected* with respect to the extension semantics $\sigma$ in $F$. We denote the set of credulously accepted, skeptically accepted and rejected arguments in AF $F$ with respect to the extension semantics $\sigma$ as $\mathrm{cred}_\sigma(F)$, $\mathrm{skep}_\sigma(F)$ and $\mathrm{rej}_\sigma(F)$, respectively.

## 3 Extension-ranking Semantics

In this section, we introduce a new kind of semantics that offers more expressiveness than binary classification. We consider preorders over sets of arguments allowing us to state that one set of arguments is "closer" to being accepted than another.

Definition 6. *Let $F = (A, R) \in \mathcal{AF}$ be an AF. An* extension ranking *on $F$ is a preorder $\sqsupseteq$ over the power set $\mathcal{P}(A)$ of all arguments. An* extension-ranking semantics *$\tau$ is a function that maps $F$ to an extension ranking $\sqsupseteq^{\tau}_{F}$ on $F$.*

For an abstract argumentation framework $F = (A, R) \in \mathcal{AF}$, an extension-ranking semantics $\tau$, an extension ranking $\sqsupseteq^{\tau}_{F}$, consider sets $E, E' \subseteq A$. If $E \sqsupseteq^{\tau}_{F} E'$, we say that $E$ is *at least as plausible to be accepted as* $E'$ with respect to $\tau$ in $F$. We introduce the usual abbreviations:

- $E$ is *strictly more plausible to be accepted than* $E'$, denoted $E \sqsupset^{\tau}_{F} E'$, if $E \sqsupseteq^{\tau}_{F} E'$ but not $E' \sqsupseteq^{\tau}_{F} E$;
- $E$ and $E'$ are *equally plausible to be accepted*, denoted $E \equiv^{\tau}_{F} E'$, if $E \sqsupseteq^{\tau}_{F} E'$ and $E' \sqsupseteq^{\tau}_{F} E$;
- $E$ and $E'$ are *incomparable*, denoted $E \asymp^{\tau}_{F} E'$, if neither $E \sqsupseteq^{\tau}_{F} E'$ nor $E' \sqsupseteq^{\tau}_{F} E$.

Since it is not always reasonable to enforce a decision between two sets of arguments, we do not necessary enforce totality for our relation, thus only utilise preorders.

Example 4. *Consider again $F_2$ from Example 3. As previously shown $E_1 = \{a\}$, $E_2 = \{a, g\}$, $E_3 = \{a, c, g\}$, and $E_4 = \{a, d, g\}$ are the complete extensions. However, all these sets are treated equally in a reasoning process based on complete semantics. Only by employing a different extension semantics, such as preferred or stable extension semantics, can these sets be differentiated in terms of their plausibility of acceptance.*

*When examining sets that are not accepted with respect to complete extension semantics, such as $\{d\}$ or $\{a, b\}$, these sets are considered equal with respect to the complete extension semantics. However, $\{d\}$ is an admissible set, whereas $\{a, b\}$ is not even conflict-free. Therefore, it is reasonable to assert that $\{d\}$ is more plausible to be accepted than $\{a, b\}$. Thus, for an extension-ranking semantics $\tau$:*

$$\{d\} \sqsupset^{\tau}_{F_2} \{a, b\}.$$

The connection between extension-ranking semantics and extension semantics can be established. In fact, extension semantics can be used directly to define a very naive instance of extension-ranking semantics, as follows:

Definition 7. *Let $F = (A, R) \in \mathcal{AF}$ be an AF. Given an extension semantics $\sigma$, we define the* least-discriminating extension-ranking semantics *wrt. $\sigma$, denoted $\mathsf{LD}^{\sigma}$ by:*

- *$E \sqsupset^{\mathsf{LD}^{\sigma}}_{F} E'$ if $E \in \sigma(F)$ and $E' \notin \sigma(F)$;*
- *and $E \equiv^{\mathsf{LD}^{\sigma}}_{F} E'$, if $E, E' \in \sigma(F)$ or $E, E' \notin \sigma(F)$.*

The least-discriminating extension-ranking semantics always gives a binary classification between $\sigma$-extensions and not $\sigma$-extensions. Thus, $\mathsf{LD}^{\sigma}$ always has only two layers, each $\sigma$-extension is ranked above every non-accepted set with respect to $\mathsf{LD}^{\sigma}$ and sets on the same layer are equally as plausible to be accepted.

Example 5. *Consider AF $F_2$ from Example 3, with the complete extensions $E_1 = \{a\}$, $E_2 = \{a, g\}$, $E_3 = \{a, c, g\}$, and $E_4 = \{a, d, g\}$. These complete extensions are equally plausible to be accepted with respect to the least-discriminating extension-ranking semantics regarding* co*, i.e.:*

$$\{a\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, g\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, c, g\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, d, g\}.$$

*All other sets are not accepted with respect to the complete extension semantics and are therefore ranked below all* co*-extensions, but these sets are ranked as equally plausible to be accepted. For example, the two rejected sets $\{d\}$ and $\{a, b\}$ are equally plausible to be accepted, i.e., $\{d\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, b\}$. Thus, we can extend the ranking from above:*

$$\{a\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, g\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, c, g\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, d, g\} \sqsupset^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{d\} \equiv^{\mathsf{LD}^{\mathrm{co}}}_{F_2} \{a, b\}.$$

Note that the least-discriminating extension-ranking semantics is reflexive and transitive for $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.

Proposition 1. $\mathsf{LD}^\sigma$ *is reflexive and transitive for* $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$. *Proof: P. 41*

To further explore the relationship between extension-ranking semantics and extension semantics, we identify the most plausible sets of an extension ranking, or the highest-ranked sets in the preorder.

Definition 8. *Let* $F = (A, R) \in \mathcal{AF}$ *be an AF. We denote by* $\mathtt{max}_\tau(F)$ *the maximal (or* most plausible*) elements of the extension ranking* $\sqsupseteq^\tau_F$*, defined as:*

$$\mathtt{max}_\tau(F) = \{E \subseteq A \mid \nexists E' \subseteq A \text{ with } E' \sqsupset^\tau_F E\}.$$

A set $E$ is among the most plausible sets with respect to an extension-ranking semantics $\tau$ in an abstract argumentation framework $F$, if there is no strictly more plausible set.

Example 6. *Continuing from Example 5, consider the least-discriminating extension-ranking semantics* $\mathsf{LD}^{\mathrm{co}}$ *with respect to the complete semantics* co*, we get:*

$$\mathtt{max}_{\mathsf{LD}^{\mathrm{co}}}(F_2) = \{\{a\}, \{a, g\}, \{a, c, g\}, \{a, d, g\}\}.$$

*Every other set is ranked below these sets, making the complete extension the most plausible sets. If we consider the least-discriminating extension-ranking semantics* $\mathsf{LD}^{\mathrm{pr}}$ *with respect to the preferred semantics* pr*, we get:*

$$\mathtt{max}_{\mathsf{LD}^{\mathrm{pr}}}(F_2) = \{\{a, c, g\}, \{a, d, g\}\}.$$

*Again the most plausible sets coincide with the* $\sigma$*-extensions.*

In this section, we introduced the notion of extension-ranking semantics, which ranks sets of arguments within an abstract argumentation framework. This ranking is more expressive than the binary classification of acceptable and non-acceptable sets induced by extension semantics. Additionally, we demonstrated the connection between extension-ranking semantics and extension semantics by defining an extension-ranking semantics based on an extension semantics.

## 4 Principles

To assess and compare extension-ranking semantics, we introduce several general principles that describe various aspects of an intuitively well-behaved extension-ranking semantics. Some of these principles are adapted from existing literature on argument-ranking semantics, while others are specifically designed for extension-ranking semantics.

### 4.1 Definitions of Principles

Classical extension semantics already possess many desirable properties for argumentative evaluation (Torre and Vesic 2017). Therefore, the formalisation of extension-ranking semantics should align with these classical extension semantics, ensuring that the constraints of acceptable and non-acceptable sets of arguments are reflected in the extension ranking. We consider two aspects:

(1) For a given extension semantics $\sigma$, we expect that only $\sigma$-extensions can be among the most plausible sets.
(2) Since different $\sigma$-extensions are indistinguishable by $\sigma$, all $\sigma$-extensions should be among the most plausible sets.

We formalise the first requirement through a principle called $\sigma$*-soundness* and the second through $\sigma$*-completeness*. Combining these two principles gives us the principle of $\sigma$*-generalisation*.

Definition 9. *Let* $\sigma$ *be an extension semantics and* $\tau$ *an extension-ranking semantics.* $\tau$ *satisfies*

- $\sigma$-soundness *if and only if for all AFs* $F$*:* $\mathtt{max}_\tau(F) \subseteq \sigma(F)$*;*
- $\sigma$-completeness *if and only if for all AFs* $F$*:* $\mathtt{max}_\tau(F) \supseteq \sigma(F)$*;*

- $\sigma$-generalisation *if and only if* $\tau$ *satisfies both* $\sigma$-soundness *and* $\sigma$-completeness.

For an extension-ranking semantics that satisfies $\sigma$-generalisation, the best-ranked sets, and thus the most plausible sets, coincide with the set of $\sigma$-extensions of the given abstract argumentation framework.

Example 7. *Consider* $F_2$ *from Example 3 and let* $\tau$ *be an extension-ranking semantics. If* $\tau$ *satisfies* co*-generalisation, then the most plausible sets are:*

$$\texttt{max}_\tau(F_2) = \{\{a\}, \{a, g\}, \{a, c, g\}, \{a, d, g\}\}.$$

*If* $\tau$ *satisfies* pr*-generalisation, then the most plausible sets are:*

$$\texttt{max}_\tau(F_2) = \{\{a, c, g\}, \{a, d, g\}\}.$$

Conflict-freeness is a desirable property in formal argumentation, as it ensures that no agent in a discussion contradicts themself. We define the principle of *respect conflicts* which states that every conflict-free set should be strictly more plausible to be accepted than conflicting ones. This principle is similar to the *self-contradiction* principle of argument-ranking semantics, where self-contradicting arguments are considered the worst arguments.

Definition 10. *Let* $\tau$ *be an extension-ranking semantics.* $\tau$ *satisfies* respect conflicts *if and only if for all AFs* $F = (A, R)$ *with* $E, E' \in A$ *it holds that* $E \in \mathrm{cf}(F)$ *and* $E' \notin \mathrm{cf}(F)$ *implies* $E \sqsupset^\tau_F E'$.

Example 8. *Consider AF* $F_2$ *from Example 3. The argument e is self-attacking, making the set* $\{e\}$ *strictly less plausible to be accepted than the grounded extension* $\{a, c, g\}$ *with respect to an extension-ranking semantics* $\tau$ *satisfying* respect conflicts*, thus:*

$$\{a, c, g\} \sqsupset^\tau_{F_2} \{e\}.$$

Respect conflicts enforces a strict separation between conflict-free and conflicting sets, ensuring that cf-*soundness* is satisfied.

Proposition 2. *If extension-ranking semantics* $\tau$ *satisfies respect conflicts, then* $\tau$ *satisfies* cf*-soundness.*

*Proof: P. 41*

Respect conflicts does not imply cf-completeness, since cf-completeness only concern the most plausible sets, while respect conflicts does not impose any relationship between two conflict-free sets, unlike cf-completeness.

Example 9. *Consider again* $F_2$ *from Example 3. Let* $\tau$ *be the extension-ranking semantics that ranks all sets as equally plausible. While* $\tau$ *satisfies* cf*-completeness, it violates respect conflicts because the conflict-free set* $\{a\}$ *is equally plausible to be accepted with the conflicting set* $\{e\}$*:*

$$\{a\} \equiv^\tau_{F_2} \{e\}.$$

*Next, consider the extension-ranking semantics* $\mathsf{LD}^{\mathrm{ad}}$*, which satisfies* cf*-completeness because every most plausible set with respect to* $\mathsf{LD}^{\mathrm{ad}}$ *is admissible and thus conflict-free. However, the conflict-free set* $\{b\}$ *and the conflicting set* $\{e\}$ *are equally plausible to be accepted*

$$\{b\} \equiv^{\mathsf{LD}^{\mathrm{ad}}}_{F_2} \{e\}.$$

*Thus, violating respect conflicts.*

*Finally, let* $\tau$ *be an extension-ranking semantics such that for sets* $E, E' \subseteq A$, $E \sqsupset^\tau_F E'$ *if* $E \in \mathrm{cf}(F)$ *and* $E' \notin \mathrm{cf}(F)$*, or if both are conflict-free and* $|E| > |E'|$*. Otherwise,* $E \equiv^{\tau'}_F E'$*. This extension-ranking semantics satisfies respect conflicts. Considering the admissible sets* $\{a, c\}$ *and* $\{a, c, g\}$*, we have:*

$$\{a, c, g\} \sqsupset^{\tau'}_{F_2} \{a, c\}.$$

*However, for* cf*-completeness, it must hold that:*

$$\{a, c, g\} \equiv^{\tau'}_{F_2} \{a, c\}.$$

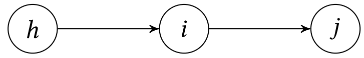


Fig. 3. AF $F_3$ from Example 10.

The plausibility of accepting a set of arguments should not be affected by arguments that are unconnected to that set. This means that adding or removing unconnected arguments should not influence the plausibility of accepting a set of arguments. A similar concept was proposed by Torre and Vesic (Torre and Vesic 2017) through the principles of *Crash-resistance* and *Non-interference* for extension semantics.

When two unconnected abstract argumentation frameworks are combined into one, the relationships between sets in the combined AF should mirror those in the original, smaller AFs. This requirement is formalised by the *composition* principle. Additionally, when an AF is split into unconnected components, the relationship between sets should remain unchanged. This requirement is captured by the *decomposition* principle.

Definition 11. *Let $\tau$ be an extension-ranking semantics.*

- *$\tau$ satisfies* composition *if for every AF $F$ that can be split into $F_1$ and $F_2$ such that $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$, $R_1 \cap R_2 = \emptyset$ and $E, E' \subseteq A_1 \cup A_2$, it holds that*
$$\text{if} \left\{ \begin{array}{l} E \cap A_1 \sqsupseteq^{\tau}_{F_1} E' \cap A_1 \\ E \cap A_2 \sqsupseteq^{\tau}_{F_2} E' \cap A_2 \end{array} \right\} \text{then } E \sqsupseteq^{\tau}_{F} E'.$$
- *$\tau$ satisfies* decomposition *if for every AF $F$ that can be split into $F_1$ and $F_2$ such that $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$, $R_1 \cap R_2 = \emptyset$ and $E, E' \subseteq A_1 \cup A_2$, it holds that*
$$\text{if } E \sqsupseteq^{\tau}_{F} E' \text{ then} \left\{ \begin{array}{l} E \cap A_1 \sqsupseteq^{\tau}_{F_1} E' \cap A_1 \\ E \cap A_2 \sqsupseteq^{\tau}_{F_2} E' \cap A_2 \end{array} \right\}.$$

Example 10. *Consider $F_2$ from Example 3 and $F_3 = (\{h, i, j\}, \{(h, i), (i, j)\})$ as depicted in Figure 3.*

*Those two abstract argumentation frameworks are disjoint, so for any extension-ranking semantics $\tau$, the relationships in $F_2 \cup F_3$ should be the same as in $F_2$ and $F_3$ individually. Consider the sets $\{a, c, g, h, j\}$ and $\{b, c, h, i\}$. The partition of $\{a, c, g, h, j\}$ is conflict-free in both $F_2$ and $F_3$ ($\{a, c, g\} \in \mathsf{cf}(F_2)$ and $\{h, j\} \in \mathsf{cf}(F_3)$), while $\{b, c, h, i\}$ has conflicts in both $F_2$ and $F_3$. Therefore, it is reasonable to say that $\{a, c, g, h, j\}$ is more plausible to be accepted than $\{b, c, h, i\}$ in both $F_2$ and $F_3$ ($\{b, c\} \notin \mathsf{cf}(F_2)$ and $\{h, i\} \notin \mathsf{cf}(F_3)$):*
$$\{a, c, g\} \sqsupset^{\tau}_{F_2} \{b, c\} \quad \text{and} \quad \{h, j\} \sqsupset^{\tau}_{F_3} \{h, i\}.$$
*For composition to be satisfied, it must hold that:*
$$\{a, c, g, h, j\} \sqsupset^{\tau}_{F_2 \cup F_3} \{b, c, h, i\}.$$
*Next, we observe that $\{a, c, g, h, j\}$ is conflict-free and $\{b, c, h, i\}$ is not conflict-free in $F_2 \cup F_3$. Thus, for any extension-ranking semantics $\tau$ that generalises $\mathsf{LD}^{\mathsf{cf}}$, we get:*
$$\{a, c, g, h, j\} \sqsupset^{\tau}_{F_2 \cup F_3} \{b, c, h, i\}.$$
*To satisfy decomposition, the relationship between $\{a, c, g, h, j\}$ and $\{b, c, h, i\}$ has to remain the same in $F_2$ and $F_3$, thus:*
$$\{a, c, g\} \sqsupset^{\tau}_{F_2} \{b, c\} \quad \text{and} \quad \{h, j\} \sqsupset^{\tau}_{F_3} \{h, i\}$$

A crucial property of abstract argumentation is *reinstatement*, as discussed by M. Caminada (M. Caminada 2006). Reinstatement involves making an attacked argument acceptable by attacking its attacker. The complete extension semantics implements this property strictly: “if an argument can be reinstated by a set of arguments, it must be included in that set”. Thus, a set of arguments that includes an argument it defends is more plausible to be accepted than a set without that argument. We define two versions of this principle: *weak reinstatement* and *strong reinstatement*. The weak version of reinstatement states that adding a defended argument (which does not introduce further conflicts) does not reduce the plausibility of acceptance of any set of arguments. The strong version ensures that adding a defended argument to a set strictly increases the plausibility of acceptance of that set.

Definition 12. *Let $\tau$ be an extension-ranking semantics.*

- *$\tau$ satisfies* weak reinstatement *if and only if for all AFs $F = (A, R)$ with $E \subseteq A$, it holds that $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^- \cup E^+)$, implies $E \cup \{a\} \sqsupseteq^\tau_F E$.*
- *$\tau$ satisfies* strong reinstatement *if and only if for all AFs $F = (A, R)$ with $E \subseteq A$, it holds that $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^- \cup E^+)$, implies $E \cup \{a\} \sqsupset^\tau_F E$.*

Note that the condition $a \notin (E^- \cup E^+)$ is necessary to prevent adding additional conflicts to the set, which could reduce the plausibility of acceptance of the set of arguments.

Example 11. *Consider $F_3$ from Example 10. The argument $j$ is defended by the argument $h$. For any extension-ranking semantics $\tau$ that satisfies weak reinstatement, it must hold that:*

$$\{h, j\} \sqsupseteq^\tau_{F_3} \{h\}.$$

*For strong reinstatement, the relationship between $\{h, j\}$ and $\{h\}$ must be strict, this means:*

$$\{h, j\} \sqsupset^\tau_{F_3} \{h\}.$$

*If we consider the set $\{h, i\}$, one might think that $\{h, i, j\} \sqsupseteq^\tau_{F_3} \{h, i\}$ should also hold, since $j$ is defended by $\{h, i\}$. However, adding $j$ to the set creates more conflicts, because $j \in \{h, i\}^+_{F_3}$. Therefore, we cannot enforce the relationship between $\{h, i, j\}$ and $\{h, i\}$. In fact, one could argue that $\{h, i\} \sqsupset^\tau_{F_3} \{h, i, j\}$ should hold, as $\{h, i\}$ has fewer conflicts than $\{h, i, j\}$.*

Argumentation is inherently non-monotonic, meaning that changes in the abstract argumentation framework can affect the conclusions drawn. However, these changes should align with intuitive reasoning. For instance, adding an attacker to an argument should not strengthen that argument. An extension-ranking semantics should reflect this behaviour, ensuring that if a set $E$ is more plausible to be accepted than another set $E'$, adding attacks from $E$ to $E'$ should not make $E$ less plausible to be accepted than $E'$. Thus, an extension-ranking semantics should be *robust* to the addition of attacks, which intuitively should not diminish the plausibility of acceptance of a set. A similar robustness concept was discussed by Rienstra, Sakama, et al. (Rienstra, Sakama, et al. 2020) for labelling-based semantics.

Definition 13. *Let $\tau$ be an extension-ranking semantics. $\tau$ satisfies* addition robustness *if for all AFs $F = (A, R)$ and sets $E, E' \subseteq A$ with $E \sqsupseteq^\tau_F E'$, it holds that $E \sqsupseteq^\tau_{F'} E'$ where $F' = (A, R')$ with $R' = R \cup \{(a, b)\}$ such that $a \in E$, $b \notin E$, and $b \in E'$.*

The principle addition robustness ensures that the relationship between two sets is not reversed by adding an attack from a more plausible set to another.

Example 12. *Consider $F_2$ from Example 3 and the sets $\{a, c, g\}$ and $\{b, c, e\}$. $\{a, c, g\}$ is the stable extension, while $\{b, c, e\}$ is not even conflict-free. Let $\tau$ be an arbitrary extension-ranking semantics such that:*

$$\{a, c, g\} \sqsupset^\tau_{F_2} \{b, c, e\}.$$

*If we create a new AF $F_2'$ by adding an attack from $a$ to $e$, this provides an additional reason to reject $\{b, c, e\}$. Therefore, if $\tau$ satisfies addition robustness, then:*

$$\{a, c, g\} \sqsupset^\tau_{F_2'} \{b, c, e\}.$$

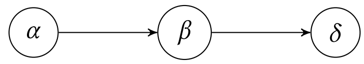

Fig. 4. AF $F_4$ from Example 13.

Table 1. Principles satisfied and violated by $\mathsf{LD}^\sigma$ for $\sigma \in \{\text{cf}, \text{ad}, \text{co}, \text{pr}, \text{gr}, \text{st}, \text{sst}\}$.

| Principles | $\mathsf{LD}^{cf}$ | $\mathsf{LD}^{ad}$ | $\mathsf{LD}^{co}$ | $\mathsf{LD}^{gr}$ | $\mathsf{LD}^{pr}$ | $\mathsf{LD}^{st}$ | $\mathsf{LD}^{sst}$ |
|---|---|---|---|---|---|---|---|
| *$\sigma$-generalisation* | ✓($\sigma$ = cf) | ✓($\sigma$ = ad) | ✓($\sigma$ = co) | ✓($\sigma$ = gr) | ✓($\sigma$ = pr) | ✗ | ✓($\sigma$ = sst) |
| *respect conflicts* | ✓ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ |
| *composition* | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| *decomposition* | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ |
| *weak reinstatement* | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| *strong reinstatement* | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ |
| *addition robustness* | ✓ | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ |
| *syntax independence* | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

In abstract argumentation, the content of each argument is disregarded, focusing solely on the relationships between arguments. It should not matter whether an argument is labelled $a$ or $b$, as long as the abstract argumentation framework's structure remains the same. Argument-ranking semantics adheres to this requirement through the principle abstraction (Amgoud and Ben-Naim 2013), which states that argument names are irrelevant to the resulting argument ranking. For extension-ranking semantics, we define a principle similar in spirit to *abstraction*, called *syntax independence*.

Definition 14. *An extension-ranking semantics $\tau$ satisfies* syntax independence *if for every pair of AFs $F = (A, R)$, $F' = (A', R')$, and for every isomorphism $\gamma : A \longrightarrow A'$, it holds for all $E, E' \subseteq A$, $E \sqsupseteq^\tau_F E'$ if and only if $\gamma(E) \sqsupseteq^\tau_{F'} \gamma(E')$.*

Example 13. *Consider again $F_3$ from Example 10 and $F_4 = (\{\alpha, \beta, \delta\}, \{(\alpha, \beta), (\beta, \delta)\})$ as depicted in Figure 4. It is easy to see that $F_4$ is isomorphic to $F_3$ and therefore, the extension rankings with respect to an extension-ranking semantics $\tau$ for these two AFs should coincide.*

## 4.2 Axiomatic Investigation of Least-Discriminating Extension-ranking Semantics

Next, we analyse the least-discriminating extension-ranking semantics with respect to the previously defined principles.

The least-discriminating extension-ranking semantics are designed to rank all $\sigma$-extensions higher than non-$\sigma$-extensions. This implies that the $\sigma$-extensions of an AF coincide with the most plausible sets with respect to $\mathsf{LD}^\sigma$. However, $\mathsf{LD}^\sigma$ usually does not satisfy $\sigma'$-generalisation for $\sigma \neq \sigma'$. For example, $\mathsf{LD}^{ad}$ does not satisfy co-soundness.

Example 14. *Consider $F_2$ from Example 3. The set $\{d\}$ is admissible, implying $\{d\} \in \mathtt{max}_{\mathsf{LD}^{ad}}(F_2)$, but $\{d\}$ is not a complete set because the argument $a$ is defended by $\{d\}$. Thus, $\mathsf{LD}^{ad}$ violates* co-soundness*. Similarly, $\mathsf{LD}^{co}$ does not satisfy* ad-completeness*. The set $\{d\}$ is admissible but not among the most plausible sets with respect to $\mathsf{LD}^{co}$, violating the requirement that $\{d\}$ should be among the most plausible sets.*

Proposition 3. *$\mathsf{LD}^\sigma$ satisfies and violates the principles as depicted in Table 1 for $\sigma \in \{\text{cf}, \text{ad}, \text{co}, \text{gr}, \text{pr}, \text{sst}\}$.*

*Proof: P. 41*

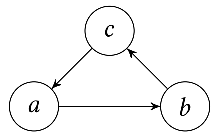


Fig. 5. AF $F_5$ from Example 15.

Note that $\mathsf{LD}^{\mathsf{st}}$ violates st-soundness. For abstract argumentation frameworks without a stable extension, there should be no most plausible set. However, $\mathsf{LD}^{\mathsf{st}}$ is still defined, though not very expressive, as every set is equally plausible to be accepted and thus among the most plausible sets.

Example 15. *Consider the odd cycle* $F_5 = (\{a, b, c\}, \{(a, b), (b, c), (c, a)\})$*, as depicted in Figure 5. This AF has no stable extension. Therefore, for any pair of sets* $E, E' \subseteq \{a, b, c\}$*. Since* $E, E' \notin \mathrm{st}(F_5)$*, it holds that:*

$$E \equiv_{F_5}^{\mathsf{LD}^{\mathsf{st}}} E'.$$

*Thus, every set is equally plausible to be accepted and considered among the most plausible sets.*

In the literature (see (Baroni, Gabbay, et al. 2018) for an overview), there are more extension semantics $\sigma$ where there are abstract argumentation frameworks without $\sigma$-extensions, thus it is impossible to define an extension-ranking semantics that satisfies $\sigma$-soundness for these extension semantics.

Proposition 4. *If* $\sigma$ *is an extension semantics such that there exists an AF* $F = (A, R)$ *with* $\sigma(F) = \emptyset$*, then no extension-ranking semantics can satisfy* $\sigma$*-soundness.* Proof: P. 46

### 4.3 Compatibilities of Principles

In Proposition 2, we have shown that respect conflicts implies cf-soundness, and in Example 9, we demonstrated that the reverse is not true. It is also evident by definition that strong reinstatement implies weak reinstatement.

While respect conflicts implies cf-soundness and strong reinstatement implies weak reinstatement, there are some incompatibilities between the principles for example ad-generalisation and strong reinstatement are incompatible.

Example 16. *Consider AF* $F_3$ *from Example 10 and the sets* $\{h\}$ *and* $\{h, j\}$*. Both sets are admissible, so for any extension-ranking semantics* $\tau$ *that satisfies* ad*-generalisation, we get:*

$$\{h\} \equiv_{F_3}^{\tau} \{h, j\}.$$

*However,* $j$ *is defended by* $\{h\}$ *and can be added to* $\{h\}$ *without creating new conflicts. Thus, strong reinstatement implies:*

$$\{h, j\} \sqsupset_{F_3}^{\tau} \{h\}.$$

*This shows that we cannot satisfies both principles simultaneously.*

For other pairs of principles ($\sigma$-generalisation, respect conflicts, composition, decomposition, weak reinstatement, strong reinstatement, addition robustness) we show that they are *pairwise independent*, meaning neither principle implies the other, but they are still compatible with each other.

Proposition 5. *The following holds:*

(1) *The principle respect conflicts implies* cf*-soundness;*
(2) *The principle strong reinstatement implies weak reinstatement;*
(3) *The principles* ad*-generalisation and strong reinstatement are incompatible;*
(4) *The principles* $\sigma$*-generalisation for* $\sigma \in \{\mathrm{co}, \mathrm{pr}, \mathrm{sst}\}$ *and addition robustness are incompatible;*

(5) *For the remaining pairs of principles $p_1, p_2 \in \{\sigma$-generalisation, respect conflicts, composition, decomposition, weak reinstatement, strong reinstatement, addition robustness$\}$ where $p_1 \neq p_2$, $p_1$ and $p_2$ are pairwise independent.* *Proof*: P. 51

In this section, we introduced several principles that a well-behaved extension-ranking semantics should satisfy. We showed that most of these principles are independent of each other. These principles provide a comprehensive overview of the behaviour of extension-ranking semantics. Additionally, we analysed the behaviour of the least-discriminating extension-ranking semantics and found that this family already satisfies many of these principles, serving as a good baseline for how an extension-ranking semantics should behave.

## 5 Base Relations

Next, we propose a general framework for defining extension-ranking semantics based on simple relations between sets of arguments. Each of these relations models a single aspect of argumentative reasoning. Individually, these relations are not very expressive, but by combining them, we can derive generalisations of various extension semantics (as we will see in the next section).

We begin with set-based relations, where two sets of arguments are compared based on the set of conflicts (Conflicts), the set of undefended arguments (Undef), the set of defended-and-not-contained arguments (Condef), or the set of unattacked arguments (Unatt) that two sets induce. In the second part of this section, we discuss cardinality-based variations of these base relations, along with other variations. The different between these two variants is that the set-based relations compare two sets utilising a subset relation, while for the cardinality-based variations the size of each computed set matter.

Before introducing the individual base relations, we define the general concept of a *base relation*. Note that a base relation is not a preorder, meaning it is not necessarily reflexive or transitive.

Definition 15. *A binary relation $\tau$ is called a* base relation *if and only if $\tau$ assigns to each $F = (A, R) \in \mathcal{AF}$ a preorder on its powerset, i.e. $\sqsupseteq^{\tau}_{F} \subseteq \mathcal{P}(A) \times \mathcal{P}(A)$.*

In other words, a base relation $\tau$ represents the relationship between two sets of arguments $E$ and $E'$ within the context of an abstract argumentation framework $F$. This allows us to determine whether $E$ is at least as plausible to be accepted as $E'$, whether the two sets are equally plausible to be accepted, or if they are incomparable.

Every extension-ranking semantics is, by definition, a base relation, even if the base relation itself is not very expressive, such as $\mathsf{LD}^{\sigma}$.

### 5.1 Set-based Base Relations

In the evaluation of abstract argumentation frameworks, extension semantics implement central notions in a binary fashion. For instance, conflict-freeness requires that extensions contain no conflicts. To achieve a more fine-grained evaluation of the plausibility of acceptance of sets of arguments, we need a more detailed assessment of these notions. Below, we define base relations that model key aspects of *conflicts*, *admissibility*, *completeness*, and *stability* by evaluating how well a given set satisfies each aspect.

The first aspect we consider is *conflicts*, the absence of which is typically seen as a desirable property in any extension semantics. Acceptable sets of arguments should be internally consistent and free from self-contradiction. Specifically, a conflict-free set of arguments is considered more plausible to be accepted than a conflicting set. Extending this idea, we regard a set $E$ to be more plausible to be accepted than another set $E'$ if $E$ has strictly fewer conflicts than $E'$ (in terms of set inclusion). This is modelled by the $\sqsupseteq^{\mathsf{Conflicts}}$ *base relation.*

Definition 16. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E \subseteq A$. Define* Conflicts *via:*

$$\mathsf{Conflicts}_F(E) = \{(a, b) \in R | a, b \in E\}.$$

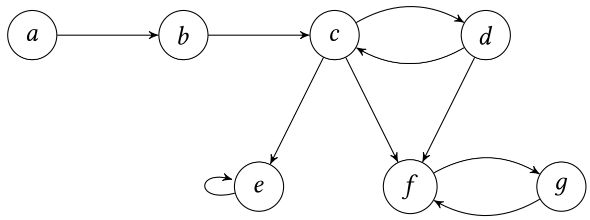


Fig. 6. Recalled AF $F_2$ from Example 3.

*The corresponding* Conflicts base relation $\sqsupseteq_F^{\mathsf{Conflicts}}$ *via:*

$$E \sqsupseteq_F^{\mathsf{Conflicts}} E' \text{ if and only if } \mathsf{Conflicts}_F(E) \subseteq \mathsf{Conflicts}_F(E').$$

Example 17. *Consider $F_2$ from Example 3 (depicted again in Figure 6). Let the sets be $\{b, c\}$ and $\{b, c, d\}$. We have* $\mathsf{Conflicts}_{F_2}(\{b, c\}) = \{(b, c)\}$ *and* $\mathsf{Conflicts}_{F_2}(\{b, c, d\}) = \{(b, c), (c, d), (d, c)\}$*. Thus, it is more plausible that $\{b, c\}$ is accepted than $\{b, c, d\}$ in terms of conflicts:*

$$\{b, c\} \sqsupset_{F_2}^{\mathsf{Conflicts}} \{b, c, d\}.$$

Another generally desirable property is *admissibility*. Sets of arguments should defend themselves against any threat. Specifically, a set of arguments that defends all its elements is considered more plausible to be accepted than a set containing at least one undefended argument. Thus, a set $E$ is more plausible to be accepted than another set $E'$ if $E$ has strictly fewer undefended arguments than $E'$ (in terms of set inclusions). This is captured by the $\sqsupseteq^{\mathsf{Undef}}$ *base relation* (Undef stands for "undefended"), where $\mathfrak{F}$ is the characteristic function.

Definition 17. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E \subseteq A$. Define* Undef *via:*

$$\mathsf{Undef}_F(E) = E \setminus \mathfrak{F}_F(E).$$

*The corresponding* Undef base relation $\sqsupseteq_F^{\mathsf{Undef}}$ *via:*

$$E \sqsupseteq_F^{\mathsf{Undef}} E' \text{ if and only if } \mathsf{Undef}_F(E) \subseteq \mathsf{Undef}_F(E').$$

Example 18. *Consider $F_2$ from Example 3 and the sets $\{b\}$ and $\{b, f\}$. Both arguments $b$ and $f$ are not defended by $\{b\}$ and $\{b, f\}$, respectively. However $\{b\}$ contains fewer undefended arguments, so:*

$$\{b\} \sqsupset_{F_2}^{\mathsf{Undef}} \{b, f\}.$$

A complete extension includes every defended argument. Thus, a set that contains all its defended arguments (without adding conflicts) is more plausible to be accepted than a set that does not include all the arguments it defends. More generally, a set $E$ is more plausible to be accepted than another set $E'$ if there are fewer arguments *consistently defended* by $E$ and not contained in $E$ than there are for $E'$ (in terms of set inclusion). With *consistent defence*, we denoted arguments that are defended by a set of arguments $E$ and neither attack $E$ or are attacked by $E$. To adequately model this notion, we need a more general notion of defence than that provided by the characteristic function $\mathfrak{F}_F$.

Example 19. *Consider AF $F_6$ depicted in Figure 7 and the set $\{b\}$. We ask:*

What arguments are defended by $\{b\}$?

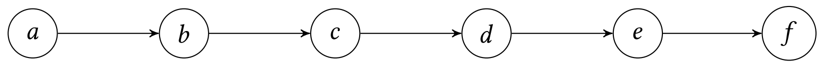


Fig. 7. AF $F_6$ from Example 19.

*Observe that $\mathfrak{F}_{F_6}(\{b\}) = \{a, d\}$. Argument $d$ is only defended because of $b$, but argument $a$ is also defended by $\{b\}$, even though it attacks $b$. Thus, we lose argument $b$ and therefore also the attack from $b$ to $c$. If we look at the defended arguments of $\{a, d\}$, we get $\mathfrak{F}_{F_6}(\{a, d\}) = \{a, c, f\}$. The defence of argument $d$ is lost. Again we have an argument $f$ whose defence depends on the acceptance of $b$, but this argument is not part of the set of defended arguments. The fixed point of this procedure is the grounded extension, $\{a, c, e\}$.*

Example 19 illustrates the need for a different definition of the characteristic function: any incoming attack on the set under examination has to be ignored. In the example, the attack from argument $a$ has to be ignored, and then we compute the defended arguments, assuming that the set $\{b\}$ is acceptable. The expected result is then $\{b, d\}$. If we iterate this notion of consistent defence, we end up with the set $\{b, d, f\}$. This set models the intuition that $d$ and $f$ are consistently defended if we assume that $\{b\}$ is accepted. We call this notion $\mathfrak{F}^*$.

Definition 18. *Let $F = (A, R) \in \mathcal{AF}$ be an AF. The function $\mathfrak{F}^*_F : \mathcal{P}(A) \to \mathcal{P}(A)$ is defined via with $i \geq 1$:*

$$\mathfrak{F}^*_F(E) = \bigcup_{i=1}^{\infty} \mathfrak{F}^*_{i,F}(E).$$

*with*

$$\mathfrak{F}^*_{1,F}(E) = E,$$
$$\mathfrak{F}^*_{i,F}(E) = \mathfrak{F}^*_{i-1,F}(E) \cup \left(\mathfrak{F}_F\left(\mathfrak{F}^*_{i-1,F}(E)\right) \setminus E^-\right).$$

With $\mathfrak{F}^*$, we get our desired results in Example 19, i. e. $\mathfrak{F}^*_{F_6}(\{b\}) = \{b, d, f\}$.

To describe $\mathfrak{F}^*$ in other words: $\mathfrak{F}^*$ is defined iteratively. We start by assuming that the set to be investigated, $E$, is admissible. We iteratively add every argument defended by $E$, and remove every attacker of $E$. Defended arguments are added until a fixed point is reached.

Proposition 6. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E \subseteq A$. Then exists a $k \in \mathbb{N}$ with:*

$$E \subseteq \mathfrak{F}^*_{1,F}(E) \subseteq \mathfrak{F}^*_{2,F}(E) \subseteq \cdots \subseteq \mathfrak{F}^*_{k-1,F}(E) \subseteq \mathfrak{F}^*_{k,F}(E) = \mathfrak{F}^*_{k+1,F}(E) = \ldots$$

*Proof: P. 53*

Next, we discuss $\mathfrak{F}^*$ further and analyse its behaviour. Since $\mathfrak{F}^*$ is a generalisation of Dung's characteristic function, these two functions should give the same results if the input is admissible.

Proposition 7. *Let $F = (A, R) \in \mathcal{AF}$ be an AF, $E \subseteq A$ such that $E \in \mathrm{ad}(F)$. Then $\mathfrak{F}_F(E) = \mathfrak{F}^*_F(E)$.* *Proof: P. 53*

A key property of the characteristic function $\mathfrak{F}_F$ is that the grounded extension is the least fixed point of that function and this property carries over to $\mathfrak{F}^*$.

Proposition 8. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E = \mathrm{gr}(F)$, then $E$ is the least fixed point of $\mathfrak{F}^*_F$.* *Proof: P. 53*

Using Proposition 7, we can show that every complete extension of an abstract argumentation framework $F$ is also a fixed point of $\mathfrak{F}^*_F$.

Proposition 9. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E \in \mathrm{co}(F)$. Then $\mathfrak{F}^*_F(E) = E$.* *Proof: P. 54*

To conclude the discussion on $\mathfrak{F}^*$, we provide a detailed example to illustrate its behaviour more thoroughly.

Example 20. *Consider $F_2$ from Example 3 and the set $\{b\}$. Argument $b$ is attacked by argument $a$, so $b^-_{F_2} = \{a\}$. To compute all consistently defended arguments by $\{b\}$, we use $\mathfrak{F}^*_{F_2}(\{b\})$. By definition, we have $\mathfrak{F}^*_{1,F_2}(\{b\}) = \{b\}$. Since arguments $a$ and $d$ are defended by $\{b\}$ and argument $b$ is not defended, we have $\mathfrak{F}_{F_2}(\{b\}) = \{a, d\}$ and therefore $\mathfrak{F}^*_{2,F_2}(\{b\}) = \{b\} \cup (\{a, d\} \setminus \{a\}) = \{b, d\}$. For the next iteration of $\mathfrak{F}^*_{F_2}$, we see that $a$, $d$ and $g$ are defended by $\{b, d\}$, $\mathfrak{F}_{F_2}(\{b, d\}) = \{a, d, g\}$. Therefore, $\mathfrak{F}^*_{3,F_2}(\{b\}) = \{b, d\} \cup (\{a, d, g\} \setminus \{a\}) = \{b, d, g\}$. Since $\{b, d, g\}$ also defends arguments $a$, $d$, and $g$, we reach a fixed point, $\mathfrak{F}^*_{4,F_2}(\{b\}) = \{b, d, g\}$. Combining each iteration, we get:*

$$\mathfrak{F}^*_{F_2}(\{b\}) = \{b, d, g\}.$$

Finally, we define the $\sqsupseteq^{\mathsf{Condef}}$ *base relation*, which models the concept of consistent defence (Condef meaning “consistently defended and not in").

Definition 19. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E \subseteq A$. We define* Condef *via:*

$$\mathsf{Condef}_F(E) = \mathfrak{F}^*_F(E) \setminus E.$$

*The corresponding* Condef base relation $\sqsupseteq^{\mathsf{Condef}}_F$ *via:*

$$E \sqsupseteq^{\mathsf{Condef}}_F E' \text{ if and only if } \mathsf{Condef}_F(E) \subseteq \mathsf{Condef}_F(E').$$

Example 21. *Continuing with Example 20, using $F_2$ from Example 3, we have already calculated $\mathfrak{F}^*_{F_2}(\{b\}) = \{b, d, g\}$, so using* Condef*, we get $\mathsf{Condef}_{F_2}(\{b\}) = \{d, g\}$. Next, we examine the set $\{b, d\}$, where $\mathfrak{F}^*_{F_2}(\{b, d\}) = \{b, d, g\}$, so $\mathsf{Condef}_{F_2}(\{b, d\}) = \{g\}$. Therefore, we have:*

$$\{b, d\} \sqsupset^{\mathsf{Condef}}_{F_2} \{b\}.$$

The final property we examine is *stability*. A set is considered stable if it attacks every argument not contained within it. To generalise this concept, we assert that a set $E$ is more plausible to be accepted than another set $E'$ if $E$ strictly attacks more arguments than $E'$ (in terms of set inclusion). We capture this aspect with the $\sqsupseteq^{\mathsf{Unatt}}$ *base relation* (Unatt stands for “unattacked”).

Definition 20. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E \subseteq A$. Define* Unatt *via:*

$$\mathsf{Unatt}_F(E) = A \setminus (E \cup E^+_F).$$

*The corresponding* Unatt base relation $\sqsupseteq^{\mathsf{Unatt}}_F$ *via:*

$$E \sqsupseteq^{\mathsf{Unatt}}_F E' \text{ if and only if } \mathsf{Unatt}_F(E) \subseteq \mathsf{Unatt}_F(E').$$

Example 22. *Consider $F_2$ from Example 3 and the two sets $\{a, c\}$ and $\{a, d\}$. Argument $b$ is attacked by $a$, and $f$ is attacked by both $c$ and $d$, while argument $g$ is not attacked by either set. Argument $e$ is only attacked by $c$. To summarise:*

$$\mathsf{Unatt}_{F_2}(\{a, c\}) = \{g\} \quad \text{and} \quad \mathsf{Unatt}_{F_2}(\{a, d\}) = \{e, g\}.$$

*This shows that :*

$$\{a, c\} \sqsupset^{\mathsf{Unatt}}_{F_2} \{a, d\}.$$

We will now delve deeper into the behaviour of the base relations. We will demonstrate that these base relations are both reflexive and transitive.

Proposition 10. *The base relation $\sqsupseteq^\tau_F$ is reflexive and transitive for $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$.*

*Proof: P. 54*

The base relations $\sqsupseteq^\tau_F$ for $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$ induce extension rankings. We are interested in investigating the satisfied and violated principled of the induced extension rankings.

Table 2. Principles satisfied and violated by $\sqsupseteq^{\tau}$ for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.

| Principles | $\sqsupseteq^{\text{Conflicts}}$ | $\sqsupseteq^{\text{Undef}}$ | $\sqsupseteq^{\text{Condef}}$ | $\sqsupseteq^{\text{Unatt}}$ |
|---|---|---|---|---|
| *σ-generalisation* | ✗ | ✗ | ✗ | ✗ |
| *respect conflicts* | ✓ | ✗ | ✗ | ✗ |
| *composition* | ✓ | ✓ | ✓ | ✓ |
| *decomposition* | ✓ | ✓ | ✓ | ✓ |
| *weak reinstatement* | ✓ | ✓ | ✓ | ✓ |
| *strong reinstatement* | ✗ | ✗ | ✓ | ✓ |
| *addition robustness* | ✓ | ✗ | ✗ | ✓ |
| *syntax independence* | ✓ | ✓ | ✓ | ✓ |

Proposition 11. *$\sqsupseteq^{\tau}$ satisfies and violates the principles as depicted in Table 2 for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.* Proof: P. 54

## 5.2 Cardinality-based Base Relations

So far we have used subset comparisons to compare sets of arguments with the base relations. However, other comparison methods are imaginable. A typical alternative to subset comparisons is *cardinality*. Thus, instead of comparing two sets based on set-inclusion, we compare them by the amount of elements these sets contain. For example, c-Conflicts compares two sets based on the number of conflicts. To avoid repeating each definition in detail, we define a generalised cardinality-based base relation $\sqsupseteq^{\text{c-}\tau}$ for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.

Definition 21. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E, E' \subseteq A$. We define c-$\tau$ for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$ as the* cardinality-based base relation *$\sqsupseteq^{\text{c-}\tau}_F$ via:*

$$E \sqsupseteq^{\text{c-}\tau}_F E' \text{ if and only if } |\tau_F(E)| \leq |\tau_F(E')|.$$

Example 23. *Consider AF $F_2$ from Example 3 and the sets $\{c, g\}$ and $\{b, f\}$. Argument $g$ is defended by $\{c, g\}$, while $c$ is not defended, and both $b$ and $f$ are not defended by $\{b, f\}$. These two sets are incomparable with respect to Undef, because $\{c\}$ is not a subset of $\{b, f\}$. However, using the cardinality-based version c-Undef, these two sets can be compared, and we get:*

$$\{c, g\} \sqsupseteq^{\text{c-Undef}}_{F_2} \{b, f\}.$$

Next, we discuss the relationship between the cardinality-based base relations and the subset-based ones. We show that the cardinality variants c-$\tau$ are generalisations of $\tau$ for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.

Proposition 12. *For AF $F = (A, R) \in \mathcal{AF}$, sets $E, E' \subseteq A$, and $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$, it holds that if $E \sqsupseteq^{\tau}_F E'$ then $E \sqsupseteq^{\text{c-}\tau}_F E'$.* Proof: P. 56

In other words, if the subset variant of a base relation can determine the relationship between two sets of arguments, then the subset variant will give us the same result. In Example 23, we saw that two sets which are incomparable with respect to $\sqsupseteq^{\text{Undef}}_{F_2}$ are comparable with respect to $\sqsupseteq^{\text{c-Undef}}_{F_2}$.

Proposition 13. *The base relation $\sqsupseteq^{\text{c-}\tau}_F$ is reflexive and transitive for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.* Proof: P. 56

Consequently, $\sqsupseteq^{\text{c-}\tau}$ induces extension rankings. Next, we examine the principles of the induced extension ranking.

Table 3. Principles satisfied and violated by $\sqsupseteq^{\text{c-}\tau}$ for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.

| Principles | $\sqsupseteq^{\text{c-Conflicts}}$ | $\sqsupseteq^{\text{c-Undef}}$ | $\sqsupseteq^{\text{c-Condef}}$ | $\sqsupseteq^{\text{c-Unatt}}$ |
|---|---|---|---|---|
| *σ-generalisation* | ✗ | ✗ | ✗ | ✗ |
| *respect conflicts* | ✓ | ✗ | ✗ | ✗ |
| *composition* | ✓ | ✓ | ✓ | ✓ |
| *decomposition* | ✗ | ✗ | ✗ | ✗ |
| *weak reinstatement* | ✓ | ✓ | ✓ | ✓ |
| *strong reinstatement* | ✗ | ✗ | ✓ | ✓ |
| *addition robustness* | ✓ | ✗ | ✗ | ✓ |
| *syntax independence* | ✓ | ✓ | ✓ | ✓ |

Proposition 14. *$\sqsupseteq_F^{\text{c-}\tau}$ satisfies and violates the principles as depicted in Table 3 for* $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$. *Proof: P. 57*

The difference between the two variants of base relations is that $\sqsupseteq^{\text{c-}\tau}$ violates decomposition for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$, while $\sqsupseteq^{\tau}$ satisfies this principle. Thus, we trade totality with decomposition.

Konieczny et al. (Konieczny et al. 2015) use comparisons of extensions to restrict the credulous or skeptical acceptance of arguments to only the "best" extensions. We extend their definitions to apply to arbitrary pairs of sets of arguments, not just $\sigma$-extensions, and define new base relations. First, we recall the concept of *strong defence*. The standard notion of defence suggests that arguments can defend themselves against their attackers. However, this self-defence has been criticised by Baroni and Giacomin (Baroni and Giacomin 2007), leading to the proposal of a stronger notion.

Definition 22. *Let $F = (A, R) \in \mathcal{AF}$ be an AF, $a \in A$, and $E \subseteq A$. An argument $a$ is* strongly defended *by $E$ (denoted as $sd_F(a, E, E')$) from $E' \subseteq A$ if and only if for every argument $b \in E'$ with $(b, a) \in R$, there is an argument $c \in E \setminus \{a\}$ such that $(c, b) \in R$ and $sd_F(c, E \setminus \{a\}, E')$.*

In other words, an argument $a$ is strongly defended by a set $E$ if $E$ contains a defender of $a$, which is not $a$, for every attacker of $a$, and each defender $c$ must be defended by $E$ without including $a$ and itself. By $sd_F(E, E')$, we refer to the strongly defended arguments by $E$ from $E'$ in abstract argumentation framework $F$. Using the notion of strong defence, extension semantics like *strongly preferred semantics* can be defined similar to Definition 4 (Baroni and Giacomin 2007; M. Caminada 2014).

Next, we define two base relations based on notions of Konieczny et al. (Konieczny et al. 2015).

Definition 23. *Let $F = (A, R) \in \mathcal{AF}$ be an AF, and $E, E' \subseteq A$ be two sets of arguments. We define $\tau \in \{\text{nonatt}, \text{strdef}\}$ as base relations $\tau$ as follows:*

- $E \sqsupseteq_F^{\text{nonatt}} E'$ *if* $|E \setminus (E_F^- \cap E')| \geq |E' \setminus (E_F'^- \cap E)|$;
- $E \sqsupseteq_F^{\text{strdef}} E'$ *if* $|sd_F(E, E')| \geq |sd_F(E', E)|$.

Intuitively, sets with more unattacked arguments are more plausible to be accepted. The nonatt relation models this idea directly for a pair of sets of arguments. If a set $E$ is attacked less by $E'$ than vice versa, it means $E$ faces fewer threats from $E'$ than vice versa, making $E$ more plausible to be accepted than $E'$. The strdef relation extends this idea by requiring strong defence instead of merely considering unattacked arguments.

Example 24. *Consider $F_2$ from Example 3. The sets $\{a, c, g\}$ and $\{a, d\}$ are both admissible sets. $\{a, c, g\}$ attacks the argument $d$, while $\{a, d\}$ only attacks $c$. This means $\{a, c, g\}$ contains more unattacked arguments than $\{a, d\}$, leading to:*

$$\{a, c, g\} \sqsupset_{F_2}^{\text{nonatt}} \{a, d\}.$$

*For* strdef*, consider the sets* $\{a, b, f\}$ *and* $\{g\}$*.* $\{g\}$ *is an admissible set, while* $\{a, b, f\}$ *is not conflict-free. However, argument* $a$ *is strongly defended by* $\{a, b, f\}$ *against* $\{g\}$*, whereas* $g$ *is not strongly defended by* $\{g\}$ *against the attack of argument* $f$*. Thus, the number of strongly defended arguments in* $\{a, b, f\}$ *is higher than in* $\{g\}$*, resulting in:*

$$\{a, b, f\} \sqsupset^{\mathsf{strdef}}_{F_2} \{g\}.$$

In Example 24, we see that $\sqsupseteq^{\mathsf{nonatt}}$ and $\sqsupseteq^{\mathsf{strdef}}$ behave differently from $\sqsupseteq^{\mathsf{Undef}}$. For $\sqsupseteq^{\mathsf{Undef}}$, we have $\{a, c, g\} \equiv^{\mathsf{Undef}}_{F_2} \{a, d\}$ and $\{g\} \sqsupset^{\mathsf{Undef}}_{F_2} \{a, b, f\}$.

Note that $\sqsupseteq^{\mathsf{nonatt}}$ and $\sqsupseteq^{\mathsf{strdef}}$ are not transitive, so these two base relations do not inherently induce extension rankings.

Example 25. *Recall* $F_5 = (\{a, b, c\}, \{(a, b), (b, c), (c, a)\})$ *from Example 15. Consider the sets* $\{a\}$*,* $\{b\}$ *and* $\{c\}$*. If we compare these sets with* nonatt*, we get:*

$$\{a\} \sqsupset^{\mathsf{nonatt}}_{F_5} \{b\} \quad \text{and} \quad \{b\} \sqsupset^{\mathsf{nonatt}}_{F_5} \{c\}.$$

*Assuming transitivity, it must hold:*

$$\{a\} \sqsupset^{\mathsf{nonatt}}_{F_5} \{c\}.$$

*However, this is false because* $c$ *is not attacked by* $a$*, while* $c$ *attacks* $a$*, leading to:*

$$\{c\} \sqsupset^{\mathsf{nonatt}}_{F_5} \{a\}.$$

*This violates transitivity, and the same behaviour applies to* $\sqsupseteq^{\mathsf{strdef}}_{F_5}$*.*

Besides violating transitivity, $\sqsupseteq^{\mathsf{nonatt}}$ and $\sqsupseteq^{\mathsf{strdef}}$ have another drawback: they ignore internal conflicts within sets, making it irrelevant whether a set is conflict-free or not. Even worse, the set containing every argument $A$ is ranked "better" than any other set.

Proposition 15. *For AF* $F = (A, R) \in \mathcal{AF}$*,* $A \sqsupseteq^{\tau}_{F} E$ *for any* $E \subset A$ *with* $\tau \in \{\mathsf{nonatt}, \mathsf{strdef}\}$*.* *Proof: P. 59*

The set $A$ is always among the most conflicting sets (for any AF with attacks) and should therefore be among the least acceptable sets, as conflict-freeness is a desirable property in formal argumentation. All $\sigma$-extensions according to Dung's extension semantics are conflict-free. Other extension semantics, such as *naive*, *stage* or *CF2* (Baroni, Gabbay, et al. 2018; Baroni and Giacomin 2003; Baroni, Giacomin, and Guida 2005; Verheij 2003, 1996) are also based on conflict-freeness.

We have not explored a subset variant of the $\sqsupseteq^{\mathsf{nonatt}}$ and $\sqsupseteq^{\mathsf{strdef}}$, since these variants do not align with their underlying motivations.

Example 26. *Consider AF* $F_2$ *from Example 3 and the sets* $\{a, c\}$ *and* $\{a, g\}$*. Neither set attacks the other, so* $\{a, c\} \setminus (\{a, c\}^-_F \cap \{a, g\}) = \{a, c\}$ *and* $\{a, g\} \setminus (\{a, g\}^-_F \cap \{a, c\}) = \{a, g\}$*. With the base relation* nonatt*, we can compare these sets. Despite sharing on argument (a), we cannot compare them using subset comparison because they are not subsets of each other.*

Example 26 highlights the limitations of a subset variation of $\sqsupseteq^{\mathsf{nonatt}}$ and $\sqsupseteq^{\mathsf{strdef}}$. Although we can easily identify the unattacked arguments in each set, the sets can still be incomparable.

In this section, we have introduced the notion of base relations to compare two sets of arguments. Each base relation models a simple aspect of argumentative reasoning within an abstract argumentation framework, whether it is comparing sets based on their conflicts, undefended arguments, consistently defended arguments, or unattacked arguments.

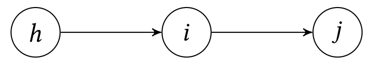


Fig. 8. Recalled AF $F_3$ from Example 10.

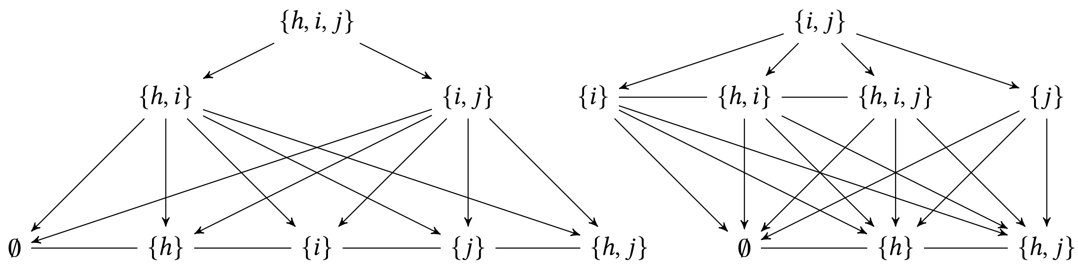


Fig. 9. Lattices depicting $\sqsupset^{\text{Conflicts}}_{F_2}$ (left) and $\sqsupset^{\text{Undef}}_{F_2}$ (right) from Example 27, where $E \longrightarrow E'$ means $E' \sqsupset^{\tau}_{F_3} E$ and $E - E'$ means $E' \equiv^{\tau}_{F_3} E$.

## 6 Combinations

The base relations from the previous section focus on individual aspects of argumentative reasoning, such as minimising conflicts or maximising self-defence. However, these base relations alone do not fully capture extension-based reasoning for abstract argumentation frameworks. To comprehensively model argumentative reasoning, we need to combine the base relations.

Example 27. *Recall $F_3$ from Example 10 (depicted again in Figure 8). Since $\{j\}$ is conflict-free and $\{h, i, j\}$ is not, we argued that $\{j\}$ is more plausible to be accepted than $\{h, i, j\}$. Using the base relations $\sqsupset^{\text{Conflicts}}_{F_3}$ gives us the intended result:*

$$\{j\} \sqsupset^{\text{Conflicts}}_{F_3} \{h, i, j\}.$$

*However, $\sqsupset^{\text{Conflicts}}$ does not fully model argumentative reasoning about this AF, since $\{j\}$ is equally as plausible to be accepted as the admissible set $\{h, j\}$ with respect to $\sqsupset^{\text{Conflicts}}_{F_3}$. We argue that this should not be the case, as $\{j\}$ is not as plausible to be accepted as any admissible set. Using $\sqsupset^{\text{Undef}}_{F_3}$ gives us the intended result of:*

$$\{h, j\} \sqsupset^{\text{Undef}}_{F_3} \{j\}.$$

*But, the sets $\{j\}$ and $\{h, i, j\}$ are incomparable with respect to* Undef. *Thus, both $\sqsupset^{\text{Conflicts}}_{F_3}$ and $\sqsupset^{\text{Undef}}_{F_3}$ alone are not sufficient to fully capture argumentative reasoning. Figure 9 depicts $\sqsupset^{\text{Conflicts}}_{F_3}$ and $\sqsupset^{\text{Undef}}_{F_3}$ as lattices.*

*By combining the two base relations of $\sqsupset^{\text{Conflicts}}_{F_3}$ and $\sqsupset^{\text{Undef}}_{F_3}$, using one base relation after another, we can model the reasoning process for $F_3$ better. Figure 10 depicts the preorder $\sqsupset^{\text{Conflicts,Undef}}_{F_3}$, where $\sqsupset^{\text{Conflicts}}_{F_3}$ is used first, and if it returns equality, $\sqsupset^{\text{Undef}}_{F_3}$ is used next.*

*The admissible sets $\emptyset$, $\{h\}$, and $\{h, j\}$ are the most plausible sets with respect to $\sqsupset^{\text{Conflicts,Undef}}_{F_3}$. Additionally, we can also state that the set $\{i\}$ is more plausible to be accepted than $\{h, i\}$. Thus, $\sqsupset^{\text{Conflicts,Undef}}_{F_3}$ models a ranking of the sets of arguments in $F_3$ based on their plausibility of acceptance with respect to admissibility.*

We propose a general framework for defining extension-ranking semantics using a sequence of base relations.

Definition 24. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $(\tau_1, \dots, \tau_n)$ a sequence of base relations. A* combination method $\Omega$ *takes $(\tau_1, \dots, \tau_n)$ and returns an extension-ranking over $\mathcal{P}(A)$.*

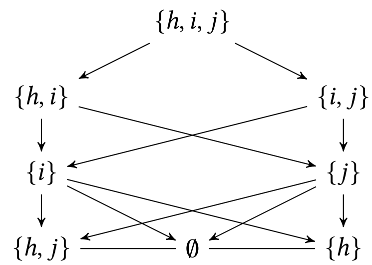


Fig. 10. $\sqsupseteq_{F_3}^{\text{Conflicts,Undef}}$ from Example 27 depicted as a lattice.

In the rest of this section, we discuss two combination methods (*Lexicographic Combination* in Section 6.1 and *Voting Methods* in Section 6.2) and analyse the resulting extension-ranking semantics.

## 6.1 Lexicographic Combination

In Example 27, we combined $\sqsupseteq^{\text{Conflicts}}$ and $\sqsupseteq^{\text{Undef}}$ by applying these two base relations sequentially. This type of combination is known as *lexicographic combination.*

Definition 25. *Let $F = (A, R) \in \mathcal{AF}$ be an AF, $E, E' \subseteq A$ and $(\sqsupseteq_F^{\tau_1}, \dots, \sqsupseteq_F^{\tau_n})$ be sequence of base relations. The* lexicographic combination $\text{lex}(\sqsupseteq_F^{\tau_1}, \dots, \sqsupseteq_F^{\tau_n})$ *is defined as follows:*

- *$E \sqsupset_F^{\text{lex}(\tau_1,\dots,\tau_n)} E'$ if and only if there exists $i$ such that $E \sqsupset_F^{\tau_i} E'$ and for all $j < i$, $E \equiv_F^{\tau_j} E'$;*
- *$E \equiv_F^{\text{lex}(\tau_1,\dots,\tau_n)} E'$ if and only if for all $i$, $E \equiv_F^{\tau_i} E'$.*

In other words, we first compare two sets of arguments according to $\tau_1$. If they are equally plausible to be accepted with respect to $\tau_1$, we move on to the next base relation $\tau_2$, continuing until we either find a difference or run out of base relations.

Example 28. *Consider again Example 27. Formally, the lattice in Figure 10 is produced by the lexicographic combination of $\sqsupseteq_{F_3}^{\text{Conflicts}}$ and $\sqsupseteq_{F_3}^{\text{Undef}}$, i.e., $\text{lex}(\sqsupseteq_{F_3}^{\text{Conflicts}}, \sqsupseteq_{F_3}^{\text{Undef}})$. For the three sets $\{h, i\}$, $\{i\}$, and $\{h, j\}$, we have:*

$$\{h, j\} \equiv_{F_3}^{\text{Conflicts}} \{i\} \sqsupset_{F_3}^{\text{Conflicts}} \{h, i\}.$$

*To differentiate between $\{h, j\}$ and $\{i\}$, we use $\sqsupseteq_{F_3}^{\text{Undef}}$ next. Therefore, we get:*

$$\{h, j\} \sqsupset_{F_3}^{\text{Undef}} \{i\}.$$

*We get the final preorder:*

$$\{h, j\} \sqsupset_{F_3}^{\text{lex(Conflicts,Undef)}} \{i\} \sqsupset_{F_3}^{\text{lex(Conflicts,Undef)}} \{h, i\}.$$

*6.1.1 Admissible Extension-ranking Semantics.* Revisiting the definition of admissibility, a set $E$ is admissible if and only if $E$ has no internal conflicts and contains no undefended argument. These two aspects are captured by Conflicts and Undef, respectively.

Definition 26. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E, E' \subseteq A$. Define the* admissible extension-ranking semantics r-ad *via:*

$$E \sqsupseteq_F^{\text{r-ad}} E' \textit{ if and only if } E \sqsupseteq_F^{\text{lex(Conflicts,Undef)}} E'.$$

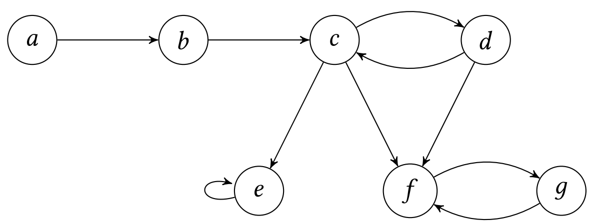


Fig. 11. Recalled AF $F_2$ from Example 3.

In other words, the set $E$ is more plausible to be accepted than $E'$ with respect to $\sqsupseteq^{\text{r-ad}}$, if $E$ is more plausible to be accepted than $E'$ with respect to $\sqsupseteq^{\text{Conflicts}}$ or, in the case of equality, $E$ is more plausible to be accepted than $E'$ with respect to $\sqsupseteq^{\text{Undef}}$ in an abstract argumentation framework $F$. We rename $\text{lex}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ to r-ad to clarify the behaviour of the preorder.

Example 29. *Consider $F_2$ from Example 3, depicted in Figure 11. When comparing the sets $\{a, b\}$, $\{b\}$, and $\{b, f\}$, we see that $\{b\}$ and $\{b, f\}$ are conflict-free, while $\{a, b\}$ is not. Using $\sqsupseteq^{\text{Conflicts}}_{F_2}$, we compare these sets using the conflicts base relation. Further comparing $\{b\}$ and $\{b, f\}$, we see that neither of these sets is admissible. Utilising $\sqsupseteq^{\text{Undef}}_{F_2}$, we get:*

$$\text{Undef}_{F_2}(\{b\}) = \{b\} \quad \textit{and} \quad \text{Undef}_{F_2}(\{b, f\}) = \{b, f\}.$$

*Thus, we get:*

$$\{b\} \sqsupset^{\text{r-ad}}_{F_2} \{b, f\}.$$

*This indicates that $\{b\}$ is closer to being admissible than $\{b, f\}$. Extending our investigation to the three sets, we get:*

$$\{b\} \sqsupset^{\text{r-ad}}_{F_2} \{b, f\} \sqsupset^{\text{r-ad}}_{F_2} \{a, b\}.$$

*For the admissible sets $\{a, c, g\}$ and $\{a, d, g\}$, we have:*

$$\text{Conflicts}_{F_2}(\{a, c, g\}) = \text{Conflicts}_{F_2}(\{a, d, g\}) = \emptyset \quad \textit{and}$$
$$\text{Undef}_{F_2}(\{a, c, g\}) = \text{Undef}_{F_2}(\{a, d, g\}) = \emptyset.$$

*This implies:*

$$\{a, c, g\} \equiv^{\text{r-ad}}_{F_2} \{a, d, g\}.$$

*Importantly, $\{a, c, g\}$ and $\{a, d, g\}$ are among the most plausible sets, $\{a, c, g\}, \{a, d, g\} \in \max_{\text{r-ad}}(F_2)$.*

*For the two conflict-free sets $\{b, g\}$ and $\{a, f\}$, we get:*

$$\text{Undef}_{F_2}(\{b, g\}) = \{b\} \quad \textit{and} \quad \text{Undef}_{F_2}(\{a, f\}) = \{f\}.$$

*Therefore, these two sets are incomparable:*

$$\{b, g\} \asymp^{\text{r-ad}}_{F_2} \{a, f\}.$$

*The full preorder of $\sqsupseteq^{\text{r-ad}}_{F_3}$ from Example 27 can be found in Figure 10. The three admissible sets $\{h, j\}$, $\emptyset$, and $\{h\}$ are the most plausible sets, with every non-admissible set ranked lower. The set $\{i\}$ is closer to being admissible than $\{h, i\}$.*

Next, we show the compliance of the admissible extension-ranking semantics with the principles defined in Section 4.

Proposition 16. r-ad *satisfies* ad*-generalisation, respect conflicts, composition, decomposition, weak reinstatement, addition robustness, and syntax independence.* r-ad *violates strong reinstatement.* *Proof: P. 59*

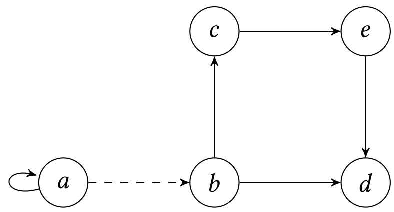


Fig. 12. AF $F_7$ from Example 31, the attack $(a, b)$ is added later to obtain $F_7'$.

For r-ad, we apply $\sqsupseteq^{\mathsf{Conflicts}}$ and then $\sqsupseteq^{\mathsf{Undef}}$ to generate the final preorder. However, if we reversed the order and apply $\sqsupseteq^{\mathsf{Undef}}$ first and then $\sqsupseteq^{\mathsf{Conflicts}}$, conflicting sets like $A$ would be ranked quite high.

Example 30. *Consider $F_2$ from Example 3. For any admissible set $E$, we still get:*

$$\mathsf{Undef}_{F_2}(\{E\}) = \emptyset \quad \textit{and} \quad \mathsf{Conflicts}_{F_2}(\{E\}) = \emptyset.$$

*So, $E$ is among the most plausible sets for* lex(Undef, Conflicts).

*If we consider the sets $\{a, b, c, d, e, f, g\}$ and $\{b, f\}$, then:*

$$\mathsf{Undef}_{F_2}(\{a, b, c, d, e, f, g\}) = \{b\} \quad \textit{and} \quad \mathsf{Undef}_{F_2}(\{b, f\}) = \{b, f\}.$$

*This implies:*

$$\{a, b, c, d, e, f, g\} \sqsupset_{F_2}^{\mathsf{lex}(\mathsf{Undef},\mathsf{Conflicts})} \{b, f\}.$$

*Although $\{b, f\}$ is conflict-free, $\{a, b, c, d, e, f, g\}$ is not, meaning that respect conflicts is violated by* lex(Undef, Conflicts).

Respect conflicts is not the only principle violated by lex(Undef, Conflicts), which is satisfied by lex(Conflicts, Undef). The principle addition robustness is also violated.

Example 31. *Consider AF $F_7 = (\{a, b, c, d, e\}, \{(a, a), (b, c), (b, d), (c, e), (e, d)\})$, as depicted in Figure 12, and the sets $\{a, d\}$ and $\{a, b, c, d\}$. We have seen that:*

$$\{a, b\} \sqsupset_{F_7}^{\mathsf{Undef}} \{a, b, c, d\}.$$

*We add the attack $(a, b)$ to create $F_7'$. However, in this AF, we get:*

$$\{a, b, c, d\} \sqsupset_{F_7'}^{\mathsf{Undef}} \{a, b\}.$$

*So, addition robustness is violated by* lex(Undef, Conflicts).

By applying the base relation $\sqsupseteq^{\mathsf{Conflicts}}$ first, we enforce that conflicting sets are less plausible to be accepted than conflict-free sets.

*6.1.2 Complete Extension-ranking Semantics.* Complete extensions are admissible sets in which every defended argument is included. A generalisation of the complete extension semantics should be based on the admissible extension-ranking semantics and refined by favouring sets that contain more defended arguments. This behaviour can be modelled with a lexicographic combination of $\sqsupseteq^{\mathsf{Conflicts}}$, $\sqsupseteq^{\mathsf{Undef}}$, and $\sqsupseteq^{\mathsf{Condef}}$.

Definition 27. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E, E' \subseteq A$. Define the* complete extension-ranking semantics r-co *via:*

$$E \sqsupseteq_F^{\text{r-co}} E' \textit{ if and only if } E \sqsupseteq_F^{\mathsf{lex}(\mathsf{Conflicts},\mathsf{Undef},\mathsf{Condef})} E'.$$

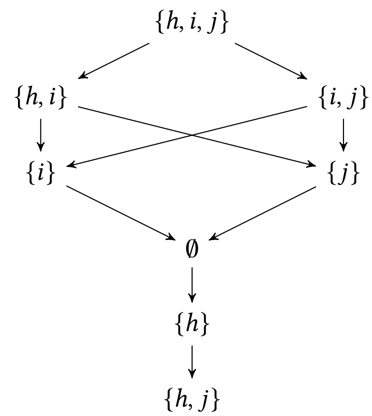


Fig. 13. $\sqsupseteq^{\text{r-co}}_{F_3}$ from Example 33 depicted as a lattice, where $E \longrightarrow E'$ means $E' \sqsupset^{\text{r-co}}_{F_3} E$.

If $E$ is strictly more plausible to be accepted than $E'$ with respect to $\sqsupseteq^{\text{r-ad}}$, then this relation also holds for r-co. If both sets are equally plausible to be accepted with respect to $\sqsupseteq^{\text{r-ad}}$, then the base relation $\sqsupseteq^{\text{Condef}}$ is used.

Example 32. *Consider $F_2$ from Example 3 and the sets $\{b\}$, $\{g\}$, and $\{d\}$. While $\{g\}$ and $\{d\}$ are both admissible, the set $\{b\}$ is not admissible. Using $\sqsupseteq^{\text{r-ad}}$, we get:*

$$\{g\} \sqsupset^{\text{r-co}}_{F_2} \{b\} \quad \textit{and} \quad \{d\} \sqsupset^{\text{r-co}}_{F_2} \{b\}.$$

*To further compare $\{g\}$ and $\{d\}$, we examine $\sqsupseteq^{\text{Condef}}$. Argument $a$ is defended by both sets, while $\{d\}$ defends $g$ in addition to $a$. Thus, we get:*

$$\text{Condef}_{F_2}(\{g\}) = \{a\} \quad \textit{and} \quad \text{Condef}_{F_2}(\{d\}) = \{a, g\}.$$

*Therefore, we get:*

$$\{g\} \sqsupset^{\text{r-co}}_{F_2} \{d\}.$$

*Since $\sqsupseteq^{\text{r-co}}$ is built on top of $\sqsupseteq^{\text{r-ad}}$, we can further extend the result of Example 29:*

$$\{g\} \sqsupset^{\text{r-co}}_{F_2} \{d\} \sqsupset^{\text{r-co}}_{F_2} \{b\} \sqsupset^{\text{r-co}}_{F_2} \{b, f\} \sqsupset^{\text{r-co}}_{F_2} \{a, b\}.$$

*Among these sets, $\{g\}$ is the closed set to be a complete extension, although it is not complete itself.*

Example 33. *Consider again $F_3$ from Example 27. The lattice in Figure 13 depicts the preorder $\sqsupseteq^{\text{r-co}}_{F_3}$. The first three levels are the same as in the preorder $\sqsupseteq^{\text{r-ad}}_{F_3}$, thus $\{h, i, j\}$ is the worst ranked set, then we have $\{h, i\}$ and $\{i, j\}$, which are incomparable with each other. After these sets, we get the two singleton sets $\{i\}$ and $\{j\}$. With $\sqsupseteq^{\text{r-co}}_{F_3}$, we can distinguish the sets $\emptyset$, $\{h\}$, and $\{h, j\}$. These were the sets for which both* Conflicts *and* Undef *returned the empty set and were therefore the most plausible sets. For $\sqsupseteq^{\text{r-co}}_{F_3}$ we get, that the complete set $\{h, j\}$ is the most plausible set, and between the two admissible sets $\emptyset$ and $\{h\}$, we can say that $\{h\}$ is closer to being complete than $\emptyset$.*

The complete extension semantics is a refinement of admissibility, thus we should refine $\sqsupseteq^{\text{r-ad}}$ with an additional base relation. In this case, we use $\sqsupseteq^{\text{Condef}}$, and this combination indeed serves as a generalisation of the complete extension semantics.

Proposition 17. r-co *satisfies* co-*generalisation, respect conflicts, composition, decomposition, weak reinstatement, strong reinstatement, and syntax independence.* r-co *violates addition robustness.* *Proof: P. 60*

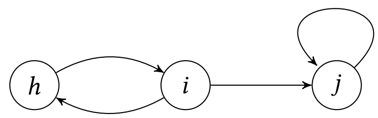


Fig. 14. AF $F_8$ from Example 35.

*6.1.3 Grounded Extension-ranking Semantics.* The grounded extension is defined as the minimal complete extension. It is natural to refine the complete extension-ranking semantics by incorporating a notion of minimality to obtain a generalisation of the grounded extension semantics. The *subset relation* $\subseteq$ directly models this notion of minimality and can be used as a base relation. In other words, a set $E$ is more plausible to be accepted than $E'$ if $E$ is smaller (with respect to the subset comparisons) than $E'$. Note that this base relation is only useful in combination with other base relations, as the structure of the underlying abstract argumentation framework is completely irrelevant for determining the plausibility of acceptance of a set with respect to $\subseteq$.

Now, we can define a generalisation of the grounded extension semantics by refining $\sqsupseteq^{\text{r-co}}$ with $\subseteq$.

Definition 28. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E, E' \subseteq A$. Define* grounded extension-ranking semantics r-gr *via:*

$$E \sqsupseteq_F^{\text{r-gr}} E' \text{ if and only if } E \sqsupseteq_F^{\text{lex(Conflicts,Undef,Condef,}\subseteq)} E'.$$

Example 34. *Consider $F_2$ from Example 3 and the sets $\{g\}$, $\{a, g\}$, and $\{a, c, g\}$. $\{a, g\}$ and $\{a, c, g\}$ are both complete extensions, while $\{g\}$ is not complete. Hence, $\sqsupseteq^{\text{r-co}}$ gives us:*

$$\{a, g\} \sqsupset_{F_2}^{\text{r-gr}} \{g\} \quad \text{and} \quad \{a, c, g\} \sqsupset_{F_2}^{\text{r-gr}} \{g\}.$$

*$\{a, g\}$ and $\{a, c, g\}$ are not the grounded extension, however $\{a, g\} \subset \{a, c, g\}$, which means:*

$$\{a, g\} \sqsupset_{F_2}^{\text{r-gr}} \{a, c, g\}.$$

*Using the results from Example 32, we can extend the ranking:*

$$\{a, g\} \sqsupset_{F_2}^{\text{r-gr}} \{a, c, g\} \sqsupset_{F_2}^{\text{r-gr}} \{g\} \sqsupset_{F_2}^{\text{r-gr}} \{d\} \sqsupset_{F_2}^{\text{r-gr}} \{b\} \sqsupset_{F_2}^{\text{r-gr}} \{b, f\} \sqsupset_{F_2}^{\text{r-gr}} \{a, b\}.$$

Example 35. *Let $F_8 = (\{h, i, j\}, \{(h, i), (i, h), (i, j), (j, j)\})$ be an AF, as depicted in Figure 14. The full preorders of $\sqsupseteq_{F_8}^{\text{r-co}}$ (left) and $\sqsupseteq_{F_8}^{\text{r-gr}}$ (right) are depicted as lattices in Figure 15. The main difference between these two preorders is that $\emptyset$ ranks better than $\{h\}$ and $\{i\}$ in $\sqsupseteq_{F_8}^{\text{r-gr}}$. Although $\{h\}$ and $\{i\}$ are not the grounded extension, these two sets are still considered more plausible to be accepted than every other set except $\emptyset$ because they are complete extensions.*

A set of arguments $E$ that is more plausible to be accepted than another set $E'$ with respect to the complete extension-ranking semantics is also more plausible to be accepted than $E'$ with respect to the grounded extension-ranking semantics. Thus, these two extension-ranking semantics mirror the behaviour of Dung's complete and grounded extension semantics.

The unique grounded extension is the smallest set among the complete extensions. Creating a preorder based on $\sqsupseteq^{\text{r-co}}$, which is then refined based on the size of the sets, entails a generalisation of the grounded extension semantics. We can infer the properties of r-gr via r-co. Since r-co satisfies respect conflicts, composition, decomposition, and strong reinstatement, we know that r-gr satisfies these principles as well, and also violates addition robustness.

Proposition 18. r-gr *satisfies* gr*-generalisation, respect conflicts, composition, decomposition, weak reinstatement, strong reinstatement, and syntax independence.* r-gr *violates addition robustness.* *Proof: P. 61*

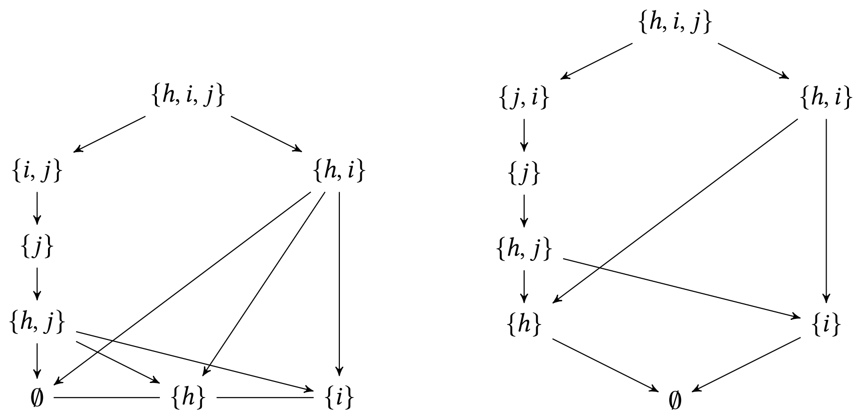


Fig. 15. Lattices depicting $\sqsupseteq^{\text{r-co}}_{F_8}$ (left) and $\sqsupseteq^{\text{r-gr}}_{F_8}$ (right) from Example 35.

*6.1.4 Preferred Extension-ranking Semantics.* The preferred extension semantics uses a maximisation approach to reason. More precisely, the preferred extensions are the maximally admissible sets. We can model this using the *superset relation* $\supseteq$, where a superset $E$ is more plausible to be accepted than its subset $E'$.

To define a generalisation of the preferred extension semantics, we lexicographically combine $\sqsupseteq^{\text{r-ad}}$ with $\supseteq$.

Definition 29. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E, E' \subseteq A$. Define* preferred extension-ranking semantics r-pr *via:*

$$E \sqsupseteq^{\text{r-pr}}_F E' \text{ if and only if } E \sqsupseteq^{\text{lex(Conflicts,Undef},\supseteq)}_F E'.$$

Example 36. *Continuing with Example 3, consider the sets $\{b\}$, $\{a\}$, and $\{a, g\}$. $\{a\}$ and $\{a, g\}$ are admissible sets, while $\{b\}$ is not admissible. Based on* r-ad*, we have:*

$$\{a\} \sqsupset^{\text{r-pr}}_{F_2} \{b\} \quad \text{and} \quad \{a, g\} \sqsupset^{\text{r-pr}}_{F_2} \{b\}.$$

*Additionally, both $\{a\}$ and $\{a, g\}$ are not preferred, but $\{a\} \subset \{a, g\}$, so:*

$$\{a, g\} \sqsupset^{\text{r-pr}}_{F_2} \{a\}.$$

*This means $\{a, g\}$ is closer to being preferred than $\{a\}$. Extending the ranking from Example 29, we get:*

$$\{a, g\} \sqsupset^{\text{r-pr}}_{F_2} \{a\} \sqsupset^{\text{r-pr}}_{F_2} \{b\} \sqsupset^{\text{r-pr}}_{F_2} \{b, f\} \sqsupset^{\text{r-pr}}_{F_2} \{a, b\}.$$

Example 37. *Consider $F_8$ from Example 35. The preorder $\sqsupseteq^{\text{r-pr}}_{F_8}$ is depicted in Figure 16. If we compare the lattice for $\sqsupseteq^{\text{r-pr}}_{F_8}$ with the lattice for $\sqsupseteq^{\text{r-co}}_{F_8}$, we see that the most plausible sets with respect to $\sqsupseteq^{\text{r-co}}_{F_8}$ are split. $\emptyset$ is ranked worse than $\{h\}$ and $\{i\}$, which aligns with the preferred extension semantics, as $\emptyset$ is not a preferred extension, while $\{h\}$ and $\{i\}$ are.*

The preferred extensions can also be characterised as maximally complete extensions. For extension semantics, this change in definition does not alter the acceptance of any set, since every preferred extension is also a complete extension (Baroni, Gabbay, et al. 2018). However, defining the preferred extension-ranking semantics based on completeness rather than admissibility yields different extension rankings.

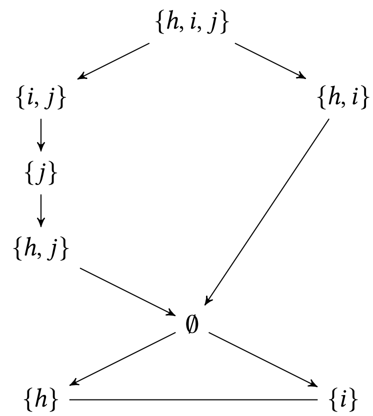


Fig. 16. Lattices depicting $\sqsupseteq_{F_8}^{\text{r-pr}}$ from Example 35.

Definition 30. *Let $F = (A, R) \in \mathcal{AF}$ and $E, E' \subseteq A$. We define the* complete-preferred extension-ranking semantics r-co-pr *via:*

$$E \sqsupseteq_F^{\text{r-co-pr}} E' \text{ if and only if } E \sqsupseteq_F^{\text{lex(Conflicts,Undef,Condef,}\supseteq)} E'.$$

Example 38. *Consider $F_2$ from Example 3 and sets $\{d\}$ and $\emptyset$. Using* r-pr*, we obtain:*

$$\{d\} \sqsupset_{F_2}^{\text{r-pr}} \emptyset.$$

*since both sets are admissible and $\{d\}$ is the larger set. However, using $\sqsupseteq^{\text{r-co-pr}}$, we get:*

$$\emptyset \sqsupset_{F_2}^{\text{r-co-pr}} \{d\}.$$

*Since these two sets are not complete, but* $\text{Condef}_{F_2}(\{d\}) = \{a, g\}$ *and* $\text{Condef}_{F_8}(\emptyset) = \{a\}$*. Switching from admissibility to completeness changes our rankings.*

*As another example, consider the sets $\{a, g\}$, $\{a, c\}$, and $\emptyset$. All three sets are admissible, so they are equally plausible to accept with respect to $\sqsupseteq_{F_2}^{\text{r-ad}}$. However, $\emptyset \subset \{a, g\}$ and $\emptyset \subset \{a, c\}$, therefore:*

$$\{a, g\} \sqsupset_{F_2}^{\text{r-pr}} \emptyset \quad \text{and} \quad \{a, c\} \sqsupset_{F_2}^{\text{r-pr}} \emptyset.$$

*But $\{a, g\}$ and $\{a, c\}$ are incomparable with respect to $\subseteq$. We get:*

$$\{a, g\} \asymp_{F_2}^{\text{r-pr}} \{a, c\} \sqsupset_{F_2}^{\text{r-pr}} \emptyset.$$

*For $\sqsupseteq_{F_2}^{\text{r-co-pr}}$, we compare $\{a, g\}$, $\{a, c\}$, and $\emptyset$ with $\sqsupseteq^{\text{Condef}}$. $\{a, g\}$ is a complete extension, so* $\text{Condef}_{F_2}(\{a, g\}) = \emptyset$*. Since* $\text{Condef}_{F_2}(\{a, c\}) = g$ *and* $\text{Condef}_{F_2}(\emptyset) = a$*, the other two sets are not complete. Among these three sets, $\{a, g\}$ is the closest one to be preferred, while the other two are incomparable.*

$$\{a, g\} \sqsupset_{F_2}^{\text{r-co-pr}} \{a, c\} \asymp_{F_2}^{\text{r-co-pr}} \emptyset.$$

Example 38 shows that r-pr and r-co-pr are not refinements of each other and produce different rankings. Despite the differences between r-pr and r-co-pr, both satisfy pr-generalisation.

Proposition 19. r-pr *and* r-co-pr *satisfy* pr-*generalisation, respect conflicts, composition, decomposition, strong reinstatement, syntax independence, and violates addition robustness.* *Proof: P. 62*

The extension-ranking semantics r-pr and r-co-pr satisfy the same principles. Therefore, depending on the context and application, either version can be used as a generalisation of the preferred extension semantics. When deciding which version to use, the importance of completeness of sets must be considered. If sets that are consistently defended and contain more arguments are to be favoured, then r-co-pr should be used; otherwise r-pr can be used instead.

*6.1.5 (Semi-)Stable Extension-ranking Semantics.* For a set of arguments to be stable, it must be conflict-free and attack all arguments not contained in that set. We have already shown in Proposition 4 that there is no extension-ranking semantics that satisfies st-soundness, and therefore st-generalisation is impossible to satisfy. Although we could consider defining an extension-ranking semantics by lexicographically combining $\sqsupseteq^{\mathsf{Conflicts}}$ and $\sqsupseteq^{\mathsf{Unatt}}$, the resulting semantics will not be st-sound. However, a generalisation of the semi-stable extension semantics is possible. Recall that a set $E$ is semi-stable if it is complete and maximises $E \cup E^{+}$. We achieve a generalisation by lexicographically combining $\sqsupseteq^{\mathsf{Conflicts}}$, $\sqsupseteq^{\mathsf{Undef}}$, $\sqsupseteq^{\mathsf{Condef}}$, and $\sqsupseteq^{\mathsf{Unatt}}$.

Definition 31. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E, E' \subseteq A$. We define the* semi-stable extension-ranking semantics r-sst *via:*

$$E \sqsupseteq_F^{\mathsf{r\text{-}sst}} E' \text{ if and only if } E \sqsupseteq_F^{\mathsf{lex}(\mathsf{Conflicts},\mathsf{Undef},\mathsf{Condef},\mathsf{Unatt})} E'.$$

Example 39. *Consider $F_2$ from Example 3 and the sets $\{a, g\}$, $\{a, d, g\}$, and $\{g\}$. $\{a, g\}$ and $\{a, d, g\}$ are both complete extensions, while $\{g\}$ is not complete. With $\sqsupseteq_{F_2}^{\mathsf{r\text{-}co}}$, we get:*

$$\{a, g\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{g\} \quad \text{and} \quad \{a, d, g\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{g\}.$$

*To further compare $\{a, g\}$ and $\{a, d, g\}$, we use $\sqsupseteq_{F_2}^{\mathsf{Unatt}}$. Both sets are not semi-stable, but:*

$$\mathsf{Unatt}_{F_2}(\{a, g\}) = \{c, d, e\} \quad \text{and} \quad \mathsf{Unatt}_{F_2}(\{a, d, g\}) = \{e\}.$$

*Thus, we get:*

$$\{a, d, g\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{a, g\}.$$

*If we compare the semi-stable extension $\{a, c, g\}$ with $\{a, d, g\}$, we get:*

$$\{a, c, g\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{a, d, g\}.$$

*because $\mathsf{Unatt}_{F_2}(\{a, c, g\}) = \emptyset$, implying $\{a, c, g\} \in \max_{\mathsf{r\text{-}sst}}(F_2)$. Since $\sqsupseteq_{F_2}^{\mathsf{r\text{-}sst}}$ is based on $\sqsupseteq_{F_2}^{\mathsf{r\text{-}co}}$, we can extend the ranking computed in Example 32:*

$$\begin{aligned}\{a, c, g\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{a, d, g\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{a, g\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{g\} \\ \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{d\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{b\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{b, f\} \sqsupset_{F_2}^{\mathsf{r\text{-}sst}} \{a, b\}.\end{aligned}$$

Example 40. *Consider $F_8$ from Example 35. The corresponding lattice to $\sqsupseteq_{F_8}^{\mathsf{r\text{-}sst}}$ is depicted in Figure 17. Comparing $\sqsupseteq_{F_8}^{\mathsf{r\text{-}sst}}$ with $\sqsupseteq_{F_8}^{\mathsf{r\text{-}co}}$, we see that these these two semantics coincide for $\{h, i, j\}$, $\{i, j\}$, $\{h, i\}$, $\{j\}$ and $\{h, j\}$. The most plausible sets of $\sqsupseteq_{F_8}^{\mathsf{r\text{-}co}}$ are refined in $\sqsupseteq_{F_8}^{\mathsf{r\text{-}sst}}$. The sets $\{h\}$ and $\{i\}$ are both more plausible to be accepted than $\emptyset$ and $\{i\}$ is more plausible to be accepted than $\{h\}$, and $\{h\}$ is more plausible to be accepted than $\emptyset$. Both $\{h\}$ and $\emptyset$ are complete extensions, but neither is a semi-stable extension. However, we show that $\{h\}$ is closer to being semi-stable than $\emptyset$.*

An important property of the semi-stable extensions is that these extensions coincide with the stable extensions as long as stable extensions exist. A similar behaviour can be observed for the semi-stable extension-ranking semantics: if stable extensions exist, then the most plausible sets of the semi-stable extension-ranking semantics are the stable extensions.

Proposition 20. *Let $F = (A, R) \in \mathcal{AF}$ be an AF with $\mathsf{st}(F) \neq \emptyset$, then $\max_{\mathsf{r\text{-}sst}}(F) = \mathsf{st}(F)$.* *Proof: P. 63*

$$
\begin{tikzcd}
& \{h, i, j\} \arrow[dl] \arrow[dr] & \\
\{i, j\} \arrow[d] & & \{h, i\} \arrow[dddl] \\
\{j\} \arrow[d] & & \\
\{h, j\} \arrow[dr] & & \\
& \emptyset \arrow[d] & \\
& \{h\} \arrow[d] & \\
& \{i\} &
\end{tikzcd}
$$

Fig. 17. Lattices depicting $\sqsupseteq^{\text{r-sst}}_{F_8}$ from Example 40.

Table 4. Principles satisfied and violated by r-$\sigma$ for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.

| Principles | r-ad | r-co | r-gr | r-pr | r-co-pr | r-sst |
|---|---|---|---|---|---|---|
| $\sigma$-generalisation | ✓($\sigma$ = ad) | ✓($\sigma$ = co) | ✓($\sigma$ = gr) | ✓($\sigma$ = pr) | ✓($\sigma$ = pr) | ✓($\sigma$ = sst) |
| respect conflicts | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| composition | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| decomposition | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| weak reinstatement | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| strong reinstatement | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ |
| addition robustness | ✓ | ✗ | ✗ | ✗ | ✗ | ✗ |
| syntax independence | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Proposition 20 also implies that r-sst satisfies st-completeness, even if the abstract argumentation framework has no stable extension.

Proposition 21. r-sst *satisfies* st-*completeness.* *Proof: P. 63*

Next, we show the satisfied and violated principles by r-sst.

Proposition 22. r-sst *satisfies* sst-*generalisation, respect conflicts, composition, decomposition, strong reinstatement, and syntax independence.* r-sst *violates addition robustness.* *Proof: P. 63*

Table 4 summarises the results of this section up to this point. For $\sigma \in \{\text{ad}, \text{co}, \text{pr}, \text{sst}\}$. Since ad-generalisation and strong reinstatement are incompatible there is no extension-ranking semantics, which satisfies a bigger set of principles than r-ad. Similar for r-$\sigma$ for $\sigma \in \{\text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$, here are $\sigma$-generalisation for $\sigma \in \{\text{co}, \text{pr}, \text{sst}\}$ and addition robustness are incompatible.

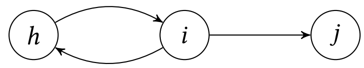


Fig. 18. AF $F_9$ from Example 42.

*6.1.6 Cardinality-based Instances.* We can utilise the cardinality-based variations of the base relations from Section 5.2 to define generalisations of extension semantics, similar to the extension-ranking semantics in the preceding sections. Instead of detailing each sequence of base relations, we will provide a summary.

Definition 32. *Let $F = (A, R) \in \mathcal{AF}$ and $E, E' \subseteq A$. The* cardinality-based extension-ranking semantics r-c-$\sigma$ *for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$ are defined as follows:*

- $E \sqsupseteq_F^{\text{r-c-ad}} E'$ *if and only if* $E \sqsupseteq_F^{\text{lex(c-Conflicts,c-Undef)}} E'$;
- $E \sqsupseteq_F^{\text{r-c-co}} E'$ *if and only if* $E \sqsupseteq_F^{\text{lex(c-Conflicts,c-Undef,c-Condef)}} E'$;
- $E \sqsupseteq_F^{\text{r-c-gr}} E'$ *if and only if* $E \sqsupseteq_F^{\text{lex(c-Conflicts,c-Undef,c-Condef,}\subseteq)} E'$;
- $E \sqsupseteq_F^{\text{r-c-pr}} E'$ *if and only if* $E \sqsupseteq_F^{\text{lex(c-Conflicts,c-Undef,}\supseteq)} E'$;
- $E \sqsupseteq_F^{\text{r-c-co-pr}} E'$ *if and only if* $E \sqsupseteq_F^{\text{lex(c-Conflicts,c-Undef,c-Condef,}\supseteq)} E'$;
- $E \sqsupseteq_F^{\text{r-c-sst}} E'$ *if and only if* $E \sqsupseteq_F^{\text{lex(c-Conflicts,c-Undef,c-Condef,c-Unatt)}} E'$.

For each base relation, the cardinality-based variant is used to generalise the corresponding extension semantics similarly to the subset comparison variant. For example, to generalise admissibility, we lexicographically combine $\sqsupseteq^{\text{c-Conflicts}}$ and $\sqsupseteq^{\text{c-Undef}}$.

Example 41. *Consider $F_2$ from Example 3 and the sets $\{c, g\}$ and $\{b, f\}$. Argument $g$ is defended by $\{c, g\}$, while $c$ is not defended, and both $b, f$ are not defended by $\{b, f\}$. $\sqsupseteq_{F_2}^{\text{r-ad}}$ returns incompatibility between these two sets, since $\{c\}$ is not a subset of $\{b, f\}$. However, using the cardinality-based variant of* r-ad, *we can compare these two sets and say that:*

$$\{c, g\} \sqsupset_{F_2}^{\text{r-c-ad}} \{b, f\}.$$

*Consider the sets $\{a, b\}$, $\{b\}$ and $\{b, f\}$. The set $\{a, b\}$ is not conflict-free, so:*

$$\{b\} \sqsupset_{F_2}^{\text{r-c-ad}} \{a, b\} \quad \text{and} \quad \{b, f\} \sqsupset_{F_2}^{\text{r-c-ad}} \{a, b\}.$$

*To further compare $\{b\}$ and $\{b, f\}$, we examine $\sqsupseteq_{F_2}^{\text{c-Undef}}$. Argument $b$ is not defended by $\{b\}$, meaning $|\text{Undef}_{F_2}(\{b\})| = |\{b\}| = 1$. For $\{b, f\}$, we have $|\text{Undef}_{F_2}(\{b, f\})| = |\{b, f\}| = 2$, which gives us:*

$$\{b\} \sqsupset_{F_2}^{\text{r-c-ad}} \{b, f\}.$$

*The resulting extension ranking between these three sets coincides with the one for $\sqsupseteq_{F_2}^{\text{r-ad}}$ as shown in Example 29.*

$$\{b\} \sqsupset_{F_2}^{\text{r-c-ad}} \{b, f\} \sqsupset_{F_2}^{\text{r-c-ad}} \{a, b\}.$$

Example 42. *Consider $F_9 = (\{h, i, j\}, \{(h, i), (i, h), (i, j)\})$ as depicted in Figure 18. The extension rankings for $\sqsupseteq_{F_9}^{\text{r-ad}}$ and $\sqsupseteq_{F_9}^{\text{r-c-ad}}$ are depicted in Figure 19. The key difference between the two rankings is that $\{h, i\}$ and $\{i, j\}$ are incomparable for $\sqsupseteq_{F_9}^{\text{r-ad}}$, whereas for $\sqsupseteq_{F_9}^{\text{r-c-ad}}$, these two sets are comparable, and $\{i, j\}$ is more plausible to be accepted than $\{h, i\}$.*

In Example 41, we observed that $\sqsupseteq_{F_9}^{\text{r-ad}}$ and $\sqsupseteq_{F_9}^{\text{r-c-ad}}$ coincide in the upper two levels. We can generalise this observation further.

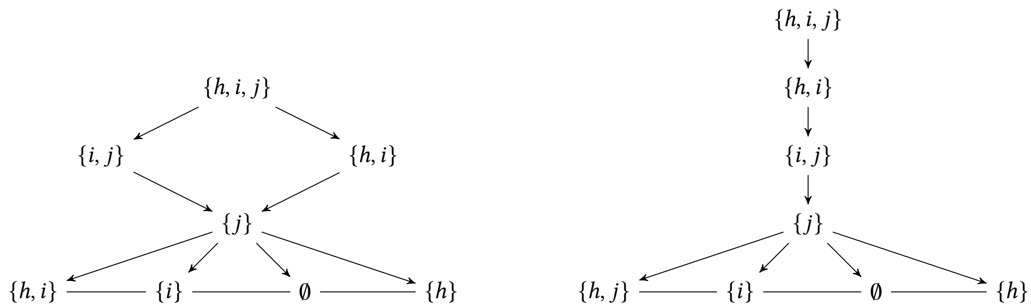


Fig. 19. Lattices depicting $\sqsupseteq^{\text{r-ad}}_{F_9}$ (left) and $\sqsupseteq^{\text{r-c-ad}}_{F_9}$ (right) from Example 42.

Table 5. Principles satisfied and violated by r-c-$\sigma$ for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.

| Principles | r-c-ad | r-c-co | r-c-gr | r-c-pr | r-c-co-pr | r-c-sst |
|---|---|---|---|---|---|---|
| $\sigma$-generalisation | ✓($\sigma$ = ad) | ✓($\sigma$ = co) | ✓($\sigma$ = gr) | ✓($\sigma$ = pr) | ✓($\sigma$ = pr) | ✓($\sigma$ = sst) |
| respect conflicts | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| composition | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| decomposition | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ |
| weak reinstatement | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| strong reinstatement | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ |
| addition robustness | ✓ | ✗ | ✗ | ✗ | ✗ | ✗ |
| syntax independence | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Proposition 23. *Let $F = (A, R) \in \mathcal{AF}$ be an AF and $E, E' \subseteq A$. If $E \sqsupseteq^{\text{r-}\sigma}_F E'$, then $E \sqsupseteq^{\text{r-c-}\sigma}_F E'$ for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.* *Proof: P. 64*

In other words, the family of extension-ranking semantics r-c-$\sigma$ is a refinement of r-$\sigma$, as it resolves the non-totality of $\sqsupseteq^{\text{r-}\sigma}$ for $\sigma \in \{\text{ad}, \text{co}, \text{sst}\}$, but coincides with $\sqsupseteq^{\text{r-}\sigma}$ when this extension-ranking semantics can compare two sets.

Next, we examine the principles that the r-c-$\sigma$ extension-ranking semantics satisfy. Starting with the $\sigma$-generalisation principle, we see that the cardinality-based variants of the extension-ranking semantics r-c-$\sigma$ are also generalisations of the extension semantics.

Proposition 24. r-c-$\sigma$ *satisfies and violates the principles as depicted in Table 5.* *Proof: P. 64*

The cardinality-based extension-ranking semantics behave similarly to their subset variants, but they trade decomposition for totality. Depending on the task at hand, we can choose the appropriate variant by evaluating between totality and decomposition.

## 6.2 Voting Methods

*Voting theory* is an area of research that explores methods and processes for collective decision-making within a group of individuals. A common application are elections, where voters collectively choose the best alternative from a set of alternatives. This selection process aims to be fair by reflecting the voters' preferences. Besides analysing voting systems, voting theory also examines paradoxes and limitations, such as the Condorcet paradox, which demonstrates that majority votes can be inconsistent. We begin by recalling essential background information on voting theory and then discuss how it can be applied to rank sets of arguments.

*6.2.1 Voting Theory.* We start with a formal definition of an *election*.

Definition 33. *An* election $(\mathcal{A}, \mathcal{N})$ *consists of a set* alternatives $\mathcal{A} = \{a_1, \ldots, a_m\}$*, and a set of voters* $\mathcal{N} = \{v_1, \ldots, v_n\}$.

To collectively decide on the alternatives in $\mathcal{A}$, each voter $i$ casts a *vote*.

Definition 34. *In an election* $(\mathcal{A}, \mathcal{N})$*, a voter* $v_i \in \mathcal{N}$ *cast a* vote *as a linear ordering* $\succsim_i$ *of* $\mathcal{A}$.

The linear order $\succsim_i$ represents the voter's preference over the alternatives, where $a \succsim_i b$ means voter $i$ prefers or votes for alternative $a$ over alternative $b$[1].

A *voting rule* determines the winner(s) of an election, the most preferred alternative based on the votes. Note that we are considering a special case of voting rules that produce a preorder rather than a single best candidate.

Definition 35. *Let* $(\mathcal{A}, \mathcal{N})$ *be an election and* $(\succsim_1, \ldots, \succsim_n)$ *the votes of* $\mathcal{N}$*. A* voting rule $\mathcal{VR}$ *returns a preorder* $\geq^{\mathcal{VR}}$ *over the alternatives* $\mathcal{A}$ *with respect to* $(\succsim_1, \ldots, \succsim_n)$.

A number of voting rules can be found in the literature, such as the *plurality voting rule*, where the alternative with the most top votes wins, or the *Borda rule*, where alternatives receive points according to their position in each vote, and the one with the highest total points wins the election. For more information on voting theory and other voting rules, we refer interested readers to (Brandt et al. 2016).

This work focuses on the *Copeland rule*, which considers the number of pairwise comparisons that an alternative wins versus the number of pairwise comparisons that the alternative loses. The alternatives with the best win/loss record are the overall winners. We choose the Copeland rule because of its simplicity and its ability to handle multiple candidates very well.

Definition 36. *Let* $(\mathcal{A}, \mathcal{N})$ *be an election and* $(\succsim_1, \ldots, \succsim_n)$ *the votes of* $\mathcal{N}$*. The* Copeland Score $Copeland(x)$ *for alternative* $x \in \mathcal{A}$ *is defined as:*

$$Copeland(x) = \sum_{i=1}^{n} |\{y \in \mathcal{A} \mid x \succsim_i y\}| - |\{y \in \mathcal{A} \mid y \succsim_i x\}|.$$

*The* Copeland Rule *Copeland for two alternatives* $x, y \in \mathcal{A}$ *is then:*

$$x \geq^{Copeland} y \text{ if and only if } Copeland(x) \geq Copeland(y).$$

Example 43. *Consider a small mayoral election with four candidates: Ali, Bea, Cleo, and Dennis. Four voters participating* $(v_1, v_2, v_3, v_4)$*, casting the votes as follows:*

$$v_1 : Ali \succ_1 Bea \succ_1 Cleo \succ_1 Dennis;$$
$$v_2 : Cleo \succ_2 Ali \succ_2 Bea \succ_2 Dennis;$$
$$v_3 : Dennis \succ_3 Cleo \succ_3 Bea \succ_3 Ali;$$
$$v_4 : Cleo \succ_4 Bea \succ_4 Dennis \succ_4 Ali.$$

[1]Note that voter's preferences in voting theory are considered strict, however for our definition, we allow weak preferences.

*Ali is ranked above Cleo by $v_1$, but below by the other three voters. Ali is ranked above Bea and Dennis by $v_1$ and $v_2$ but below by $v_3$ and $v_4$, resulting in a Copeland Score of $Copeland(Ali) = -2$. The Copeland Scores for the other candidates are:*

$$Copeland(Bea) = 0 \quad Copeland(Cleo) = 6 \quad Copeland(Dennis) = -4.$$

*The final ranking with respect to the Copeland Rule is:*

$$Cleo \geq^{Copeland} Bea \geq^{Copeland} Ali \geq^{Copeland} Dennis.$$

*Thus, Cleo is the winner of the election.*

6.2.2 *Copeland-based Combination.* A voting rule takes a number of votes, presented as linear orders, and returns a preorder over the alternatives. The input-output structure is analogous to extension-ranking semantics, where a sequence of base relations is used to construct a preorder over the powerset of arguments. In this context, the sequence of base relations serves as the votes, and the powerset of arguments represent the alternatives in an election. Thus, a voting rule can be seen as an combination method, forming the basis for a general family of extension-ranking semantics.

Definition 37. *Let $F = (A, R) \in \mathcal{AF}$ be an AF, $E, E' \subseteq A$ and $(\sqsupseteq_F^{\tau_1}, \dots, \sqsupseteq_F^{\tau_n})$ being a sequence of base relations. Consider $(\mathcal{P}(A), \{\tau_1, \dots, \tau_n\})$ as an election and $\mathcal{VR}$ as a voting rule. We define the $\mathcal{VR}$-*based combination *$\mathcal{VR}(\sqsupseteq_F^{\tau_1}, \dots, \sqsupseteq_F^{\tau_n})$ as follows:*

$$E \sqsupseteq_F^{\mathcal{VR}(\tau_1,\dots,\tau_n)} E' \text{ if and only if } E \geq^{\mathcal{VR}} E'.$$

Note that our votes are binary relations rather than linear orders. Therefore, for voting rules requiring totality and antisymmetry, we must define how to handle these cases separately.

Inspired by the Copeland rule, Konieczny et al. (Konieczny et al. 2015) proposed methods for comparing $\sigma$-extensions using *pairwise comparison criteria*, where a $\sigma$-extension $E$ is favoured over $E'$ if $E$ has a better win/loss record than $E'$ with respect to these criteria. We generalise these definition to introduce the *Copeland*-based combination as an instance of the $\mathcal{VR}$-based combination.

Definition 38. *Let $F = (A, R) \in \mathcal{AF}$ be an AF, $E, E' \subseteq A$ and $(\sqsupseteq_F^{\tau_1}, \dots, \sqsupseteq_F^{\tau_n})$ be a sequence of base relations. We define the* Copeland-based combination $\text{cope}(\sqsupseteq_F^{\tau_1}, \dots, \sqsupseteq_F^{\tau_n})$ *via:*

$$E \sqsupseteq_F^{\text{cope}(\tau_1,\dots,\tau_n)} E' \text{ if and only if } E \geq^{Copeland} E'.$$

In other words, a set $E$ is more plausible to be accepted than $E'$ if the number of sets ranked worse than $E$ is greater than those ranked worse than $E'$. This means $E$ has a better balance of wins and losses than $E'$ with respect to the selected base relations.

Example 44. *Consider $F_3$ from Example 10 and the sets $\{h, i\}$ and $\{i\}$. $\{h, i\}$ wins only against $\{h, i, j\}$ with respect to $\sqsupseteq_{F_3}^{\text{Conflicts}}$ and loses against five other sets $(\{h\}, \{i\}, \{j\}, \{h, j\}, \emptyset)$, resulting in a value of $-4$, while $\{i\}$ receives a value of 3. Thus, we get:*

$$\{i\} \sqsupset_{F_3}^{\text{cope(Conflicts)}} \{h, i\}.$$

*For $\sqsupseteq_{F_3}^{\text{Undef}}$, both $\{h, i\}$ and $\{i\}$ receive a value of $-2$, leading to:*

$$\{h, i\} \equiv_{F_3}^{\text{cope(Undef)}} \{i\}.$$

*The base relations individually rank these two sets differently. Combining these scores, we get $-6$ for $\{h, i\}$ and 1 for $\{i\}$, resulting in:*

$$\{i\} \sqsupset_{F_3}^{\text{cope(Conflicts,Undef)}} \{h, i\}.$$

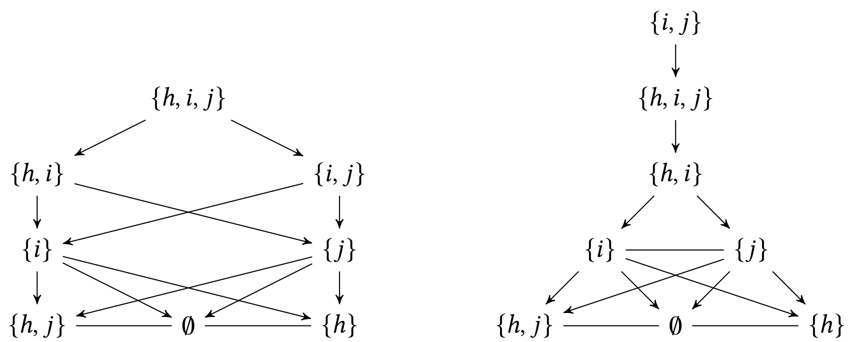


Fig. 20. Lattices depicting $\sqsupseteq^{\text{r-ad}}_{F_3}$ (left) and $\sqsupseteq^{\text{cope(Conflicts,Undef)}}_{F_3}$ (right) from Example 44.

*The full extension ranking of* $\sqsupseteq^{\text{cope(Conflicts,Undef)}}_{F_3}$ *is depicted as a lattice in Figure 20 on the right. Comparing this extension ranking with* $\sqsupseteq^{\text{r-ad}}_{F_3}$. *We observe, the set containing every argument* $\{h, i, j\}$ *is more plausible to be accepted than* $\{i, j\}$ *with respect to* $\sqsupseteq^{\text{cope(Conflicts,Undef)}}_{F_3}$, *whereas for* $\sqsupseteq^{\text{r-ad}}_{F_3}$, $\{h, i, j\}$ *is the least plausible set.*

*The two different combination methods induce different extension rankings.*

The extension-ranking semantics $\sqsupseteq^{\text{cope(nonatt)}}$ and $\sqsupseteq^{\text{cope(strdef)}}$ are refinements of the *Copeland-based extensions* $\text{CBE}_{\sigma,\gamma}$ defined by Konieczny et al. 2015.

Definition 39 ((Konieczny et al. 2015)). *Let* $F = (A, R) \in \mathcal{AF}$ *be an AF,* $\sigma$ *an extension semantics, and* $\gamma \in \{\text{nonatt}, \text{strdef}\}$. *The set of* Copeland-based extensions $\text{CBE}_{\sigma,\gamma}$ *is defined as:*

$$\text{CBE}_{\sigma,\gamma} = argmax_{E \in \sigma(F)} |\{E' \in \sigma(F) \mid E \sqsupseteq^{\gamma}_{F} E'\}| - |\{E' \in \sigma(F) \mid E' \sqsupseteq^{\gamma}_{F} E\}|.$$

Copeland-based extensions are the $\sigma$-extensions that rank highest among all $\sigma$-extensions with respect to $\gamma \in \{\text{nonatt}, \text{strdef}\}$. The extension-ranking semantics $\sqsupseteq^{\text{cope}(\gamma)}$ can compare all possible sets in $\mathcal{P}(A)$, not only the $\sigma$-extensions.

Although the relations $\sqsupseteq^{\text{nonatt}}$ and $\sqsupseteq^{\text{strdef}}$ are not transitive, $\sqsupseteq^{\text{cope(nonatt)}}$ and $\sqsupseteq^{\text{cope(strdef)}}$ are transitive and reflexive. We can generalise this observation:

Proposition 25. *Let* $F = (A, R) \in \mathcal{AF}$ *be an AF.* $\sqsupseteq^{\text{cope}(\tau_1,\ldots,\tau_n)}_{F}$ *is reflexive and transitive for any sequence of base relations* $(\tau_1, \ldots, \tau_n)$. Proof: P. 66

While the lexicographic combination lex fully preserves the input relation, when applied to a single base relation, for any abstract argumentation framework $F = (A, R)$ it holds that $\text{lex}(\sqsupseteq^{\tau}_{F}) = \sqsupseteq^{\tau}_{F}$, this behaviour does not always hold for cope.

Example 45. *Consider* $F_2$ *from Example 3 and sets* $\{b, d\}$ *and* $\{c, f\}$. *Argument* $b$ *is not attacked by* $\{c, f\}$, *while* $d$ *attacks both* $c$ *and* $f$, *meaning:*

$$\{b, d\} \sqsupset^{\text{nonatt}}_{F_2} \{c, f\}.$$

*However, using* cope, *we get:*

$$\{c, f\} \sqsupset^{\text{cope(nonatt)}}_{F_2} \{b, d\}.$$

*Changing the relationship between* $\{b, d\}$ *and* $\{c, f\}$.

*Next, consider the sets $\{b, f\}$ and $\{c, f\}$. Arguments $b$ and $f$ are strongly defended by $\{b, f\}$ against $\{c, f\}$, while $c$ and $f$ are not strongly defended by $\{c, f\}$ against $\{b, f\}$, so:*

$$\{b, f\} \sqsupset^{\mathsf{strdef}}_{F_2} \{c, f\}.$$

*However, using* cope*, we get:*

$$\{c, f\} \sqsupset^{\mathsf{cope(strdef)}}_{F_2} \{b, f\}.$$

*Again altering the relationship between the two sets.*

Example 46. *Consider $F_3$ from Example 10 and the sets $\{h, i\}$ and $\{i, j\}$. These two sets are incomparable with respect to* Conflicts*, i. e.:*

$$\{h, i\} \asymp^{\mathsf{Conflicts}}_{F_3} \{i, j\}.$$

*However, using* cope*, these two sets are comparable and equally plausible:*

$$\{h, i\} \equiv^{\mathsf{cope(Conflicts)}}_{F_3} \{i, j\}.$$

*Using* cope *we can now compare two previously incomparable sets, but the relationship between these two sets has changed.*

The two examples demonstrate that cope is not an identity function for every base relation. We know that $\sqsupseteq^{\mathsf{nonatt}}$, $\sqsupseteq^{\mathsf{strdef}}$, and $\sqsupseteq^{\mathsf{Conflicts}}$ lack important properties, namely transitivity ($\sqsupseteq^{\mathsf{nonatt}}$, $\sqsupseteq^{\mathsf{strdef}}$) and totality ($\sqsupseteq^{\mathsf{Conflicts}}$). If the underlying base relation $\tau$ satisfies these two properties, then cope($\sqsupseteq^{\tau}$) coincides with $\sqsupseteq^{\tau}$.

Proposition 26. *Let $F = (A, R) \in \mathcal{AF}$ be an AF. If base relation $\sqsupseteq^{\tau}$ is total and transitive, then* $\mathsf{cope}(\sqsupseteq^{\tau}_F) = \sqsupseteq^{\tau}_F$. *Proof: P. 66*

Next, we explore how the combination method cope complies with our principles when applied to simple sequences of base relations.

Proposition 27. $\mathsf{cope}(\sqsupseteq^{\mathsf{Conflicts}}, \sqsupseteq^{\mathsf{Undef}})$*,* $\mathsf{cope}(\sqsupseteq^{\mathsf{nonatt}})$ *and* $\mathsf{cope}(\sqsupseteq^{\mathsf{strdef}})$ *satisfy and violate the principles as depicted in Table 6.* *Proof: P. 66*

Table 6. Principles satisfied and violated by $\mathsf{cope}(\sqsupseteq^{\mathsf{Conflicts}}, \sqsupseteq^{\mathsf{Undef}})$, $\mathsf{cope}(\sqsupseteq^{\mathsf{nonatt}})$ and $\mathsf{cope}(\sqsupseteq^{\mathsf{strdef}})$.

| Principles | $\mathsf{cope}(\sqsupseteq^{\mathsf{Conflicts}}, \sqsupseteq^{\mathsf{Undef}})$ | $\mathsf{cope}(\sqsupseteq^{\mathsf{nonatt}})$ | $\mathsf{cope}(\sqsupseteq^{\mathsf{strdef}})$ |
|---|---|---|---|
| $\sigma$-generalisation | ✓($\sigma$ = ad) | ✗ | ✗ |
| respect conflicts | ✗ | ✗ | ✗ |
| composition | ✗ | ✗ | ✗ |
| decomposition | ✗ | ✗ | ✗ |
| weak reinstatement | ✓ | ✓ | ✓ |
| strong reinstatement | ✗ | ✓ | ✓ |
| addition robustness | ✗ | ✗ | ✗ |
| syntax independence | ✓ | ✓ | ✓ |

Throughout this section, we have combined previously defined base relations to construct extension-ranking semantics and analysed their behaviour by examining the principles they satisfy. We have demonstrated that when subset-based base relations are combined lexicographically, there is no extension-ranking semantics that satisfies a bigger set of principles than the resulting extension-ranking semantics. Using the Copeland-based combination allows us to generalise Dung's extension semantics. However, this approach comes at the cost of violating other desired principles.

## 7 Conclusion and Related Work

In this paper, we presented a general framework for ranking sets of arguments based on their plausibility of acceptance such that we can say that one set is "closer" to being acceptable than another set. For this purpose, we introduced extension-ranking semantics, which are functions that induce a preorder over the powerset of arguments, allowing us to say that set $E$ is more plausible to be accepted than $E'$. We also introduced a number of principles for evaluating extension-ranking semantics, and also for guiding the development of new extension-ranking semantics.

In order to define a general framework for the development of extension-ranking semantics, we considered several central aspects of argumentative reasoning and defined these aspects as simple base relations, each of which models one of these aspects. By combining these base relations, we propose a family of extension-ranking semantics, which turn out to be generalisations of Dung's classical extension semantics.

If we compare the extension-ranking semantics discussed on the basis of the principles satisfied, we see that $\mathsf{LD}^\sigma$, r-$\sigma$, and r-c-$\sigma$ all satisfy their respective $\sigma$-generalisation principles. In addition, cope($\sqsupseteq^{\mathsf{Conflicts}}, \sqsupseteq^{\mathsf{Undef}}$) satisfies ad-generalisation, showing that the proposed base relations $\sqsupseteq^{\mathsf{Conflicts}}, \sqsupseteq^{\mathsf{Undef}}, \sqsupseteq^{\mathsf{Condef}}$, and $\sqsupseteq^{\mathsf{Unatt}}$ do actually model our intended behaviour.

### 7.1 Related Work

Several works have addressed the problem of refining reasoning in abstract argumentation. Konieczny et al. (Konieczny et al. 2015) focus on refining the acceptance conditions of individual arguments by restricting when a set of arguments is considered acceptable. Bonzon et al. (Bonzon et al. 2018) and Yun et al. (Yun et al. 2018) use argument-ranking semantics to refine extension semantics in order to restrict the acceptance of a set of arguments. These works restrict their refinement on only acceptable sets and completely ignore non-acceptable sets. However, this restriction is not always recommended. For some semantics, such as the stable semantics of Dung, there are AFs without any acceptable set. Nevertheless, we wish to reason with these AFs. While these works present approaches to restricting sets, neither of them attempt to rank sets based on their plausibility of acceptance. Furthermore, neither approach distinguishes between two not acceptable sets.

Another area, where the "best" extensions are selected, is that of *preference-based argumentation frameworks* (PAF) (Alfano, Greco, et al. 2025; Amgoud and Cayrol 2002, 1998; Amgoud and Vesic 2011, 2014; Cyras 2016; Kaci et al. 2018; Silva et al. 2020). These frameworks are argumentation frameworks that have been extended with a preference over the arguments, given by an agent. Here, the *lifting-based* approach aims to select a subset of extensions of an AF that are considered the "best" extensions according to some semantic notions. There are two key differences between the research on PAFs and our work. Firstly, in the case of PAFs, the preference relations are part of the input and are based on the agents' opinions rather than being computed based on the AF alone. Secondly, our work focuses not only on the extensions, but also on general sets of arguments, providing a broader perspective.

The labelling-based semantics are a variation of extension semantics, where each argument gets a label of in, out, undec. If an argument is labelled in it means that it is part of an $\sigma$-extension, while out means that the argument is attacked by an accepted argument and is therefore definitely not part of the extension. undec (short for undecided) means that no clear decision can be found. Based on these labels, an extension can be constructed by taking every argument labelled as in. The papers (Arieli and Rienstra 2014), (Rienstra 2014) and (Rienstra and Thimm 2018) considered ordering over labellings. The latter two papers discussed some general principles for these orders, namely *conditional directionality* and *SCC stratification*, which are the counterparts of the *directionality* and *SCC decomposability* principles used in abstract argumentation (Baroni, M. Caminada, et al. 2011). These approaches can be extended to define a new family of extension-ranking semantics.

In this paper, we generalised the extension-based approaches of Dung. While these approaches are well-known and heavily researched, in the area of formal argumentation we can find a different family of reasoning approaches, the *argument-based approaches*. Instead of focusing on the sets of arguments, these approaches rank the individual arguments based on their strength. Thus, we can say that an argument $a$ is "stronger" than argument $b$, see (Amgoud and Ben-Naim 2013; Bonzon et al. 2016a, 2023) for overviews. The argument-based approaches and extension-ranking semantics share the goal of ranking elements, while argument-based approaches rank individual objects, extension-ranking semantics rank sets of objects. In (Bengel et al. 2025), the authors discussed how to transform a ranking over sets of arguments into a ranking over arguments. They present necessary and sufficient conditions the transformation and the extension ranking have to satisfy in order to for the resulting argument ranking to have certain properties. The opposite direction, i.e., transforming a ranking over arguments into a ranking over set of arguments, was discussed in (Skiba 2026). Thus, these two works have shown, that these two formalisms are related. However, we have to note that we cannot apply these two transformation one after another, as the resulting ranking would be a flat ranking.

Already Aristotle categorises arguments into *logic*, *dialectic*, and *rhetoric* types (Eemeren et al. 2014). Logical strength of an argument can be divided into two parts *inferential* and *contextual* strength. The inferential strength is concerned with how well the premise of an argument supports its conclusion. Contextual strength is concerned with how well the conclusion is supported in the context. With dialectic strength we identify how attackable an argument is in the context of a discussion. The final notion of rhetorical strength is concerned with how well an argument can persuade an agent. Prakken's recent works (Prakken 2024, 2022, 2021, 2023) discuss these strength notions, arguing that formal argumentation typically focuses on contextual strength (within the abstract argumentation framework). Prakken discusses what dialectical strength looks like in formal argumentation. The author presents an argument-ranking semantics and investigates the properties of this semantics. It turns out that the principles from Section 4 are only concerned with the contextual strength of arguments. In this paper, we interpret the strength of arguments and sets of arguments as contextual, since this is the most appropriate interpretation in our context. The strength of arguments and sets of arguments should be evaluated in the context of an abstract argumentation framework.

### 7.2 Future Work

In Example 30, we have shown that the order between $\sqsupseteq^{\mathsf{Conflicts}}$ and $\sqsupseteq^{\mathsf{Undef}}$ is important for the lexicographic combination, i.e., first applying $\sqsupseteq^{\mathsf{Conflicts}}$ ensures that respect conflicts is not violated. However, we have not discussed any other orders. Does applying $\sqsupseteq^{\mathsf{Condef}}$ before $\sqsupseteq^{\mathsf{Undef}}$ entail a different variation of the complete extension-ranking semantics? An in-depth investigation of different orders will be done in future work. However, note that for the Copeland-based combination the order of the base relation is not relevant. The extension-ranking semantics $\sqsupseteq^{\mathsf{r\text{-}pr}}$ and $\sqsupseteq^{\mathsf{r\text{-}co\text{-}pr}}$ have shown that different combinations of base relations can yield to a generalisation of the preferred extension semantics. It is interesting to check whether we can find more such examples, where different combinations of base relation yield to the same generalisations.

Besides the extension semantics discussed in this paper, there are lot more semantics such as the *stage* (Verheij 2003, 1996) or *ideal* (M. Caminada and Pigozzi 2011) extension semantics. Defining generalisations of these extension semantics is another interesting avenue for future work.

An application of extension-ranking semantics is *extension enforcement* (Baumann, Doutre, et al. 2021; Coste-Marquis et al. 2015; Haret et al. 2018; Wallner 2018, 2020; Wallner, Niskanen, et al. 2016, 2017). The goal is to modify the abstract argumentation framework so that a selected set becomes a $\sigma$-extension in the modified AF. Thimm (Thimm 2024) suggested that a set of arguments "close" to being stable might be easier to enforce than a less "close" set. Extension-ranking semantics can model this notion of closeness, with better-ranked sets being "closer" to acceptance. Investigating whether the assumption of Thimm 2024 holds is an interesting direction for future research.

Extension-ranking semantics allow us to state that a set of arguments is "closer" to being acceptable than another set of arguments. This notion of closeness can help agents in a debate or persuasion case to find their optimal positions. Some times it is not possible for an agent have a optimal strategy, thus they try to find the "closest" strategy to a optimal strategy in a debate. A investigation of the application of extension-ranking semantics in a debate or persuasion setting will be done in future work.

Our proposed base relations can help identify reasons for the non-acceptance of sets of arguments. For instance, if the base relation Undef highlights an argument $a$ for a set $E$, it indicates that $E$ does not defend $a$, explaining why $E$ cannot be admissible. This aligns with the growing interest in explanations for AI approaches, known as *XAI* (Adadi and Berrada 2018; Guidotti et al. 2019; Samek et al. 2017). Explanations of formal argumentation have also gained attention (Alfano, Calautti, et al. 2020, 2023; Baumann and Ulbricht 2018; Booth, M. Caminada, et al. 2014; Borg and Bex 2021a, 2022, 2021b; Cyras et al. 2021; Fan and Toni 2014, 2015; Potyka et al. 2023; Sakama 2018; Saribatur et al. 2020; Seselja and Straßer 2013; Timmer et al. 2015). It would be valuable to compare explanations based on our base relations with other explanation approaches and explore how extension rankings can provide explanations for settings with nonsensical constraints, offering more informative answers than mere rejection.

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# A Technical Proofs

## A.1 Technical Proofs for Section 3

**Proposition 1.** $\mathsf{LD}^\sigma$ *is reflexive and transitive for* $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.

Proof. Let $F = (A, R)$ be an AF, $E \subseteq A$, and $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$ an extension semantics.

**reflexive** By definition, $E \equiv_F^{\mathsf{LD}^\sigma} E'$ for $E = E'$. This holds because if $E \in \sigma(F)$, then $E' \in \sigma(F)$ and if $E \notin \sigma(F)$, then $E' \notin \sigma(F)$.

**transitive** Consider $E', E'' \subseteq A$ such that $E \sqsupseteq_F^{\mathsf{LD}^\sigma} E'$ and $E' \sqsupseteq_F^{\mathsf{LD}^\sigma} E''$. If $E \notin \sigma(F)$, we know that $E' \notin \sigma(F)$ and $E'' \notin \sigma(F)$, thus $E \sqsupseteq_F^{\mathsf{LD}^\sigma} E''$. If $E \in \sigma(F)$, there is no set ranked strictly better than $E$ with respect to $\mathsf{LD}^\sigma$, hence $E \sqsupseteq_F^{\mathsf{LD}^\sigma} E''$. □

## A.2 Technical Proofs for Section 4

**Proposition 2.** *If extension-ranking semantics* $\tau$ *satisfies respect conflicts, then* $\tau$ *satisfies* cf*-soundness.*

Proof. Let $\tau$ be an extension-ranking semantics that satisfies *respect conflicts*, and let $F = (A, R)$ be an AF with $E \subseteq A$ and $E \in \mathtt{max}_\tau(F)$. Since $\emptyset$ is always conflict-free and we know that $E \sqsupseteq_F^\tau \emptyset$, it follows that $E$ has to be conflict-free as well. Otherwise, respect conflicts would be violated.

Now, consider $E' \notin \mathrm{cf}(F)$, then respect conflicts implies $\emptyset \sqsupset_F^\tau E'$ and therefore $E' \notin \mathtt{max}_\tau(F)$. Thus, we conclude that $\mathtt{max}_\tau(F) \subseteq \mathrm{cf}(F)$, satisfying cf-soundness. □

**Proposition 3.** $\mathsf{LD}^\sigma$ *satisfies and violates the principles as depicted in Table 1 for* $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{sst}\}$.

Proof. We consider each principle as follows.

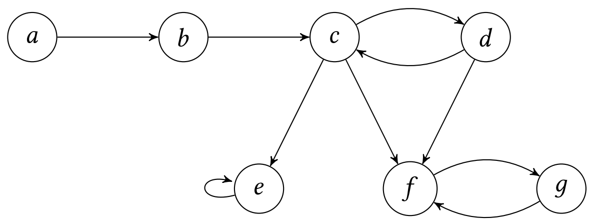


Fig. 21. Recalled AF $F_2$ from Example 3.

Lemma A.1. $\mathsf{LD}^\sigma$ *satisfies $\sigma$-generalisation for* $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{sst}\}$.

Proof. Let $F = (A, R)$ be an AF and $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{sst}\}$. For every set $E \in \sigma(F)$, by definition, $E \sqsupset_F^{\mathsf{LD}^\sigma} E'$ for every $E' \notin \sigma(F)$. Since $\mathsf{LD}^\sigma$ produces only a binary classification, this implies that every $\sigma$-extension is among the most plausible sets, and every non-$\sigma$-extension is not among the most plausible sets. Hence, $\sigma$-generalisation is satisfied. □

Lemma A.2. $\mathsf{LD}^\sigma$ *violates respect conflicts for* $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{sst}\}$.

Example 47. *Consider $F_2$ (recalled in Figure 21 and the sets $\{b\}$ and $\{e\}$. Both these sets are not admissible, but $\{b\}$ is conflict-free, while $\{e\}$ is not. For $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}, \mathrm{sst}\}$, we get:*

$$\{b\} \equiv_{F_2}^{\mathsf{LD}^\sigma} \{e\}.$$

*Therefore,* $\mathsf{LD}^\sigma$ *violates respect conflicts for* $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.

Lemma A.3. $\mathsf{LD}^{\mathrm{cf}}$ *satisfies respect conflicts.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ with $E \in \mathrm{cf}(F)$, and $E' \notin \mathrm{cf}(F)$. By definition, $E \sqsupset_F^{\mathsf{LD}^{\mathrm{cf}}} E'$. □

Lemma A.4. $\mathsf{LD}^\sigma$ *satisfies composition for* $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.

Proof. Let $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$ be an AF. To prove the theorem for $E, E' \subseteq A_1 \cup A_2$, it must hold that if

$$\left\{ \begin{array}{l} E \cap A_1 \sqsupseteq_{F_1}^{\mathsf{LD}^\sigma} E' \cap A_1 \\ E \cap A_2 \sqsupseteq_{F_2}^{\mathsf{LD}^\sigma} E' \cap A_2 \end{array} \right\} \text{ then } E \sqsupseteq_F^{\mathsf{LD}^\sigma} E'.$$

We examine each semantics case by case, starting with conflict-freeness and then admissibility. Since $\sqsupseteq^{\mathsf{LD}^\sigma}$ is only a binary classification, it is sufficient to show that if $E \cap A_i$ satisfies $\sigma$ in $F_1$ and $F_2$ for $i \in \{1, 2\}$, then it also satisfies $\sigma$ in $F$ and if $E \cap A_i$ violates $\sigma$ in $F_1$ or $F_2$ for $i \in \{1, 2\}$ then $E$ also violates $\sigma$ in F. Since if $E$ satisfies $\sigma$ in $F$, then there cannot be any set $S$ ranked strictly better than $E$. If $E \cap A_i$ violates $\sigma$ in either $F_1$ or $F_2$ and $E \cap A_1 \sqsupseteq_{F_1}^{\mathsf{LD}^\sigma} E' \cap A_1$ or $E \cap A_2 \sqsupseteq_{F_2}^{\mathsf{LD}^\sigma} E' \cap A_2$, then $E' \cap A_i$ violates $\sigma$ in $F_1$ or $F_2$ aswell for $i \in \{1, 2\}$. So, if both $E$ and $E'$ are violating $\sigma$ in $F$, then neither of these sets can be ranked strictly better.

**"$\sigma$ = cf":** If $E \cap A_1$ and $E \cap A_2$ are conflict-free in $F_1$ and in $F_2$, then $E$ is conflict-free in $F$, since the union of $F_1$ and $F_2$ does not introduce new conflicts.
If $E \cap A_1$ or $E \cap A_2$ is not conflict-free in $F_1$ or $F_2$, the responsible conflict $(a, b) \in R_{\{1,2\}}$ with $a, b \in E$ is also present of $F$. Hence, $E$ is not conflict-free in $F$.

**"$\sigma$ = ad":** If $E \cap A_1$ and $E \cap A_2$ are admissible in $F_1$ and in $F_2$, then every argument in $E$ is defended in $F$. Otherwise, there has to be an argument attacking an argument in $E$ in $F$ that is not counterattacked, however this attacker has to exists in either $F_1$ or $F_2$ as well, implying that $E \cap A_i$ can no longer be admissible in the respective $F_i$ for $i \in \{1, 2\}$. Since, $E$ also has to be conflict-free, $E$ is admissible in $F$.

W.l.o.g. assume $E \cap A_1$ is not admissible in $F_1$, then there exists an argument $a \in E$ for which there is no defender in $E$ and since $F_1$ and $F_2$ are disjoint argument $a$ remains not defended in $F$. Thus, $E$ is not admissible in $F$.

**"$\sigma = \text{co}$":** Assume $E \cap A_1$ and $E \cap A_2$ are complete in $F_1$ and $F_2$. If $E$ is not complete in $F$ there has to be an argument, which is defended by $E$ and not contained. However, this argument must exists in either $F_1$ or $F_2$ as well, making $E \cap A_i$ no longer complete in the respective $F_i$ for $i \in \{1, 2\}$.

W.l.o.g. assume $E \cap A_1$ is not complete in $F_1$, then there exists an argument $a$ which is defended by $E$ and not contained in $E$. However, since $F_1$ and $F_2$ are disjoint this argument remains defended in $F$ by $E$. Thus, $E$ cannot be complete in $F$.

**"$\sigma = \text{gr}$":** Assume $E \cap A_1$ and $E \cap A_2$ are grounded in $F_1$ and $F_2$, $E$ is the least fixed point of the characteristic function in both these AFs. Since $F_1$ and $F_2$ are disjoint, every argument defended in $F_1$ and $F_2$ is also defended in $F$. So, $E$ is a fixed point of the characteristic function in $F$. Since, the set of unattacked arguments in $F$ is the same as the union of unattacked arguments in $F_1$ and $F_2$, $E$ is minimal, proving that $E$ is grounded in $F$.

W.l.o.g. assume $E \cap A_1$ is not grounded in $F_1$, then there is a set $S \subset E$, i. e., there is an argument $a \in E$ and $a \notin S$, but $S \cup \{a\}$ defends $a$. This argument $a$ is also part of $F$, since $F = F_1 \cup F_2$ holds as well as $F_1$ and $F_2$ are disjoint. Thus, $E$ is not minimal in $F$ and therefore, not grounded.

**"$\sigma = \text{pr}$":** Assume $E \cap A_1$ and $E \cap A_2$ are preferred in $F_1$ and $F_2$, $E$ is admissible in $F$. If $E$ is not preferred in $F$, then there has to be a preferred set $S \subseteq A_1 \cup A_2$ such that $E \subset S$. Meaning there is an argument $a \in S$ and $a \notin E$, which is defended by $S$ and $E$. However, $a$ has to exits in either $F_1$ or $F_2$ and therefore $E \cap A_i$ is not maximal in that $F_i$ for $i \in \{1, 2\}$.

W.l.o.g. assume $E \cap A_1$ is not preferred in $F_1$, so there is a preferred set $S \subseteq A_1$ such that $E \subset S$. Meaning there is an argument $a \in S$ and $a \notin E$, which is defended by $S$ and $E$. Since $F_1$ and $F_2$ are disjoint, $a$ must be defended by $E$ in $F$, therefore $E$ cannot be maximal in $F$.

**"$\sigma = \text{st}$":** Assume $E \cap A_1$ and $E \cap A_2$ are stable in $F_1$ and $F_2$, if $E$ is not stable in $F$, then there has to be an argument $a \in A_1 \cup A_2$ that is not attacked by $E$ and $a \notin E$. However, this $a$ has to exits in $F_1$ or $F_2$ making $E \cap A_i$ not stable in that $F_i$ for $i \in \{1, 2\}$.

W.l.o.g. assume $E \cap A_1$ is not stable in $F_1$, there must be an argument $a \in A_1$ such that $a \notin E \cup E^+$. However, $a$ has to exists in $F$, since $F = F_1 \cup F_2$ and $F_1$ and $F_2$ are disjoint.

**"$\sigma = \text{sst}$":** Assume $E \cap A_1$ and $E \cap A_2$ are semi-stable in $F_1$ and $F_2$, $E$ is complete in $F$. If $E$ is not semi-stable in $F$, then there must be a semi-stable set $S \subseteq A_1 \cup A_2$ such that $E \subset S$. Meaning there is an argument $a \in S \cup S^+_F$ and $a \notin E$, which is attacked by $S$ or in $S$. However, $a$ has to exits in either $F_1$ or $F_2$ and therefore $E \cap A_i$ cannot be semi-stable in that $F_i$ for $i \in \{1, 2\}$.

W.l.o.g. suppose $E \cap A_1$ is not semi-stable in $F_1$, so there is a semi-stable set $S \subseteq A_1$ such that $E \subset S$. Meaning there is an argument $a \in S \cup S^+_{F_1}$ and $a \notin E \cup E^+_{F_1}$. Since $F_1$ and $F_2$ are disjoint, $a$ cannot be part of $E \cup E^+_F$ in $F$, therefore $E \cup E^+_F$ cannot be maximal in $F$. □

Lemma A.5. $\mathsf{LD}^\sigma$ *violates decomposition for* $\sigma \in \{\text{cf}, \text{ad}, \text{co}, \text{gr}, \text{pr}, \text{st}, \text{sst}\}$.

Example 48. *Let* $F_{10} = (\{a, b, c, d\}, \{(a, b), (c, d)\})$ *be an AF and the two sets* $\{a, b, c\}$ *and* $\{a, c, d\}$, *as depicted in Figure 22.*

$F_{10}$ *can be partitioned into two disjoint AFs:* $F_{10,1} = (\{a, b\}, \{(a, b)\})$ *and* $F_{10,2} = (\{c, d\}, \{(c, d)\})$. *The sets* $\{a, b, c\}$ *and* $\{a, c, d\}$ *are not conflict-free in* $F_{10}$, *so for any extension semantics based on conflict-freeness,* $\sigma \in \{\text{cf}, \text{ad}, \text{co}, \text{gr}, \text{pr}, \text{st}, \text{sst}\}$, *it holds that:*

$$\{a, b, c\} \equiv^{\mathsf{LD}^\sigma}_{F_{10}} \{a, c, d\}.$$

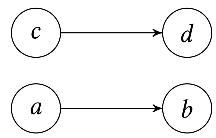


Fig. 22. AF $F_{10}$ from Example 48.


Fig. 23. AF $F_{11}$ from Example 49.

*However, $\{a, c, d\} \cap \{a, b\}$ is conflict-free in $F_{10,1}$, and $\{a, b, c\} \cap \{c, d\}$ is conflict-free in $F_{10,2}$. Thus:*

$$\{a, c, d\} \cap \{a, b\} \sqsupset^{\mathsf{LD}^\sigma}_{F_{10,1}} \{a, b, c\} \cap \{a, b\} \quad \textit{and} \quad \{a, b, c\} \cap \{c, d\} \sqsupset^{\mathsf{LD}^\sigma}_{F_{10,2}} \{a, c, d\} \cap \{c, d\}.$$

*for $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$. Thus, $\mathsf{LD}^\sigma$ violates decomposition for $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.*

Lemma A.6. *$\mathsf{LD}^\sigma$ satisfies weak reinstatement for $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$ any set. Assume $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^- \cup E^+)$. We will show that $E \not\sqsupset^{\mathsf{LD}^\sigma}_F E \cup \{a\}$. Since $\sqsupseteq^{\mathsf{LD}^\sigma}$ is a binary classification, we only need to check for $E \in \sigma(F)$ and $E \notin \sigma(F)$.

(1) If $E \notin \sigma(F)$: Since $E \notin \mathtt{max}_{\mathsf{LD}^\sigma}(F)$, there is no set $S \subseteq A$ such that $E \sqsupset^{\mathsf{LD}^\sigma}_F S$.
(2) If $E \in \sigma(F)$: Then $E \in \mathtt{max}_{\mathsf{LD}^\sigma}(F)$. For $\sigma' \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$, adding $a$ to $E$ does not introduce any conflict, so $E \cup \{a\}$ remains acceptable with respect to $\sigma$. If $E \in \mathrm{gr}(F)$, then $E = \mathfrak{F}_F(E)$, meaning there cannot exists an argument $a \in A$ such that $a \in \mathfrak{F}_F(E)$ and $a \notin E$. Thus, every $\mathsf{LD}^\sigma$ satisfies weak reinstatement. □

Lemma A.7. *$\mathsf{LD}^\sigma$ violates strong reinstatement for $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.*

Example 49. *Let $F_{11} = (\{a, b\}, \{(a, a)\})$ be an AF, as depicted in Figure 23, consider $E = \{a\}$. Since $E$ is not conflict-free, $E \notin \sigma(F_{11})$ for $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}, \mathrm{sst}\}$. However, $b \in \mathfrak{F}_{F_{11}}(E)$, $b \notin E$ and $b \notin (E^- \cup E^+)$. Based on strong reinstatement, it would imply*

$$E \cup \{b\} \sqsupset^{\mathsf{LD}^\sigma}_{F_{11}} E.$$

*but $E \cup \{b\}$ is not conflict-free. Hence, $E \cup \{b\} \notin \sigma(F_{11})$, and therefore $E \cup \{b\} \notin \mathtt{max}_{\mathsf{LD}^\sigma}(F_{11})$ and:*

$$E \cup \{b\} \equiv^{\mathsf{LD}^\sigma}_{F_{11}} E.$$

*Thus, $\mathsf{LD}^\sigma$ does violates strong reinstatement for $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}, \mathrm{sst}\}$.*

Lemma A.8. *$\mathsf{LD}^\sigma$ satisfies addition robustness for $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{gr}, \mathrm{st}\}$.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ with $a \in E$, and $b \in E' \setminus E$. We extend $F$ to $F'$ such that $F' = (A, R \cup \{(a, b)\})$. To show that $\mathsf{LD}^\sigma$ satisfies addition robustness, it is enough to show that if $E \in \sigma(F)$, then $E \in \sigma(F')$, and if $E' \notin \sigma(F)$, then $E' \notin \sigma(F')$. Since $\sqsupseteq^{\mathsf{LD}^\sigma}_F$ is a binary classification, this implies if $E$ satisfies $\sigma$ in $F'$, then $E$ is ranked among the best sets with respect to $\sqsupseteq^{\mathsf{LD}^\sigma}_{F'}$ and $E'$ cannot be ranked strictly better then $E$. Additionally, if $E' \notin \sigma(F')$, then $E$ cannot be ranked strictly worse than $E'$ with respect to $\sqsupseteq^{\mathsf{LD}^\sigma}_{F'}$.

**"$\sigma = \mathrm{cf}$":** Let $E \in \mathrm{cf}(F)$, then $E \in \mathrm{cf}(F')$ since no conflict are added to $E$. Thus, for any set $E'' \subseteq A$, $E \sqsupseteq^{\mathsf{LD}^{\mathrm{cf}}}_{F'} E''$. Let $E' \notin \mathrm{cf}(F)$, then no conflicts are removed in $F'$ with respect to $E'$. Thus, for any set $E'' \subseteq A$, $E'' \sqsupseteq^{\mathsf{LD}^{\mathrm{cf}}}_{F'} E'$. Therefore, if $E$ is conflict-free this set stays conflict-free in $F'$ and if a set is not conflict-free, then this set stays not conflict-free. Hence, the relationship between $E$ and $E'$ is unchanged with respect to $\sqsupseteq^{\mathsf{LD}^{\mathrm{cf}}}_F$.

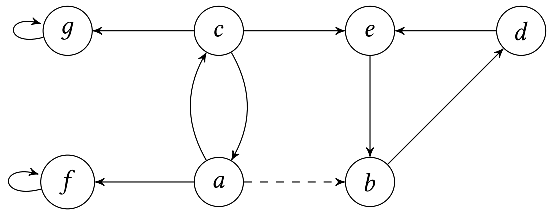


Fig. 24. AF $F_{12}$ from Example 50, where the dashed attack between $a$ and $b$, $(a, b)$, is added later to obtain $F'_{12}$.

**"$\sigma = \mathrm{ad}$":** Let $E \in \mathrm{ad}(F)$, $E$ is conflict-free in $F'$. We need to show that every argument in $E$ is defended in $F'$. Assume $a' \in E$ is not defended in $F'$ by $E$, there is at least one attacker of $a'$ that is not defeated in $F'$ and we do not remove any attack, therefore the added attack $(a, b)$ has to be the reason. Implying $a' = b$, which is impossible since $b \notin E$. Hence, $E$ has to be an admissible set in $F'$.
Let $E' \notin \mathrm{ad}(F)$ and assume $E' \in \mathrm{ad}(F')$, there is one argument $b' \in E'$ which is not defended in $F$ by $E'$ but defended in $F'$. Let $c$ be the attacker of $b'$, then $E'$ does not contain any attacker of $c$ in $F$, hence the addition of $(a, b)$ will create a new attacker for $c$ implying $c = b$ and $a \in E'$, which implies that $E'$ is not conflict-free in $F'$ and therefore also not an admissible set. If $E$ is an admissible set in $F$ it remains an admissible set and if $E'$ is not an admissible set $E'$ remains not an admissible set implying that the relationship between $E$ and $E'$ is unchanged in $F'$, i.e., $E \sqsupseteq^{\mathsf{LD}^{\mathrm{ad}}}_{F'} E'$.

**"$\sigma = \mathrm{gr}$":** Assume $E' \in \mathrm{gr}(F')$ and $E' \notin \mathrm{gr}(F)$, since $E'$ is not empty there must to be an unattacked argument $c \in E'$ and since $a \in b^-_{F'}$, $c \neq b$. Additionally, for every $c \neq a$, $c^+_F = c^+_{F'}$, hence every argument defended by $c$ in $F'$ is also defended by $c$ in $F$. Since $E' \in \mathrm{gr}(F')$, every argument in $E'$ is part of the least fixed point of $\mathfrak{F}_{F'}(\{c\})$ and everything defended by $\{c\}$ in $F'$ is also defended by $\{c\}$ in $F$, therefore every argument, which is part of the least fixed point of $\mathfrak{F}_{F'}(\{c\})$ also has to be part of the least fixed of $\mathfrak{F}_F(\{c\})$, implying that $E' \in \mathrm{gr}(F)$ contradicting the assumption.
Let $E \in \mathrm{gr}(F)$, since the grounded extension is unique, it holds that $E' \notin \mathrm{gr}(F)$ and therefore $E' \notin \mathrm{gr}(F')$. Hence, it is impossible that $E' \sqsupset^{\mathsf{LD}^{\mathrm{gr}}}_{F'} E$.

**"$\sigma = \mathrm{st}$":** Let $E \in \mathrm{st}(F)$, then $E \in \mathrm{ad}(F)$ and $E \in \mathrm{ad}(F')$. Additionally, no attack is removed by switching over to $F'$, $E^+_F \subseteq E^+_{F'}$, and $E \cup E^+_F = A$. Hence, it has to hold that $E \cup E^+_{F'} = A$, implying $E \in \mathrm{st}(F')$.
Next, we show that if $E' \notin \mathrm{st}(F)$, then $E' \notin \mathrm{st}(F')$. For contradiction assume $E' \in \mathrm{st}(F')$. If $E' \notin \mathrm{cf}(F)$ or $E' \notin \mathrm{ad}(F)$, we are done. Let $E' \in \mathrm{ad}(F)$ and therefore $E' \in \mathrm{ad}(F')$. There exist an argument $b' \in E'^+_{F'}$ such that $b' \notin E'^+_F$ and additionally it holds that $E'^+_F \subset E'^+_{F'}$. Since, the only attack we add is $(a, b)$, implying $b = b'$ and $a \in E'$. However, this also implies that $E'$ is no longer conflict-free in $F'$ since $b \in E'$. Therefore, $b'$ cannot exist and therefore $E' \notin \mathrm{st}(F')$. □

Lemma A.9. *$\mathsf{LD}^\sigma$ violates addition robustness for $\sigma \in \{\mathrm{co}, \mathrm{pr}, \mathrm{sst}\}$.*

Example 50. *Let $F_{12}$ be the AF depicted in Figure 24. Consider the sets $\{a\}$ and $\{b, c\}$, both of which are complete, preferred and semi-stable. If we add the attack $(a, b)$, then argument $d$ is defended by $\{a\}$, making $\{a\}$ not longer complete, preferred, and semi-stable. Meanwhile, $\{b, c\}$ remains complete, preferred and semi-stable. This shows that:*

$$\{a\} \sqsupseteq^{\mathsf{LD}^\sigma}_{F_{12}} \{b, c\} \quad \textit{but} \quad \{b, c\} \sqsupset^{\mathsf{LD}^\sigma}_{F'_{12}} \{a\}.$$

*for $\sigma \in \{\mathrm{co}, \mathrm{pr}, \mathrm{sst}\}$, violating addition robustness.*

Lemma A.10. *$\mathsf{LD}^\sigma$ satisfies syntax independence for $\sigma \in \{\mathrm{cf}, \mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{st}, \mathrm{sst}\}$.*

Proof. Clear by definition. □


Fig. 25. AF $F_{11}$ from Example 49.

□

**Proposition 4.** *If $\sigma$ is an extension semantics such that there exists an AF $F = (A, R)$ with $\sigma(F) = \emptyset$, then no extension-ranking semantics can satisfy $\sigma$-soundness.*

Proof. Let $\sigma$ be an extension semantics and $F = (A, R)$ an AF such that $\sigma(F) = \emptyset$. Let $\tau$ be an extension-ranking semantics. For $\tau$ to satisfy $\sigma$-soundness, it must hold that $\mathtt{max}_\tau(F) \subseteq \sigma(F)$. However, for any preorder there always exists a maximal set, meaning $\mathtt{max}_\tau(F) \neq \emptyset$. Since $\sigma(F) = \emptyset$, $\sigma$-soundness is always violated. □

To show the pairwise independence between pairs of principles, we partially introduce some new extension-ranking semantics. We start by recalling *argument-ranking semantics*, for more information we refer to (Amgoud and Ben-Naim 2013; Bonzon et al. 2016b).

Definition 40. *An* argument-ranking semantics *$\rho$ is a function, which maps an AF $F = (A, R)$ to a preorder $\succeq^\rho_F$ on $A$.*

Definition 41. *Let $F = (A, R)$ be an AF and $a \in A$. A* h-categoriser function $\mathrm{cat} : A \longrightarrow (0, 1]$ *is a function that satisfies:*

$$\mathrm{cat}(a) = \frac{1}{1 + \sum_{b \in a^-_F} \mathrm{cat}(b)}.$$

*We define the* h-categoriser argument-ranking semantics *(Cat) as:*

$$a \succeq^{Cat}_F b \text{ if and only if } \mathrm{cat}(a) \geq \mathrm{cat}(b).$$

Definition 42. *Let $F = (A, R)$ be an AF. Given an extension semantics $\sigma$, we define a variation of the* least-discriminating extension-ranking semantics *with respect to $\sigma$, denoted $\mathsf{LD}^{\sigma *}$, by:*

$$\begin{aligned}
&E \sqsupset^{\mathsf{LD}^{\sigma *}}_F E' \text{ if } E \in \sigma(F) \text{ and } E' \notin \sigma(F);\\
&E \asymp^{\mathsf{LD}^{\sigma *}}_F E' \text{ if } E, E' \notin \sigma(F);\\
&\text{and } E \equiv^{\mathsf{LD}^{\sigma *}}_F E', \text{ if } E, E' \in \sigma(F).
\end{aligned}$$

Lemma A.11. $\mathsf{LD}^{\sigma *}$ *satisfies $\sigma$-generalisation for $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{gr}, \mathrm{pr}, \mathrm{sst}\}$ and violates weak reinstatement.*

Proof. Let $F = (A, R)$ be an AF and $\sigma$ an an extension semantics. Consider $E \subseteq A$ such that $E \in \sigma(F)$. Then $E$ is ranked better than any non-extension and ranked equal to every $\sigma$-extension, so $E \in \mathtt{max}_{\mathsf{LD}^{\sigma *}}(F)$.

Consider AF $F_{11}$ from Example 49 and recalled in Figure 25. Suppose $E = \{a\}$, then $\{a\}$ is not conflict-free, implying $\{a\} \notin \sigma(F_{11})$ for $\sigma \in \{\mathrm{ad}, \mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}, \mathrm{sst}\}$, and $b \in \mathfrak{F}_{F_{11}}(\{a\})$, $b \notin \{a\}$ and $b \notin (\{a\}^- \cup \{a\}^+)$. However, $\{a, b\}$ is not conflict-free, thus:

$$\{a\} \asymp^{\mathsf{LD}^{\sigma *}}_{F_{11}} \{a, b\}.$$

Therefore, $\mathsf{LD}^{\sigma *}$ violates weak reinstatement. □

Definition 43. *Let $F = (A, R)$ be an AF, $\rho$ an argument-ranking semantics, and $E, E' \subseteq A$. We define* Kelly's extension-ranking semantics $\mathsf{Kelly}_\rho$ *via:*

(1) *If $E = E'$ then $E \equiv^{\mathsf{Kelly}_\rho}_F E'$;*

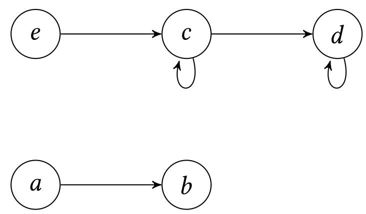

Fig. 26. AF $F_{13}$ from Lemma A.12.

(2) *If* $E = \emptyset$ *and* $E' \neq \emptyset$ *then* $E \sqsupset_F^{\mathsf{Kelly}_\rho} E'$;

(3) *Otherwise* $E \sqsupseteq_F^{\mathsf{Kelly}_\rho} E'$ *if and only if* $a \succ_F^\rho b$ $\forall a \in E$ *and* $\forall b \in E'$.

Lemma A.12. $\mathsf{lex}(\sqsupseteq^{\mathsf{c\text{-}Conflicts}}, \sqsupseteq^{\mathsf{Kelly}_{\mathsf{cat}}})$ *satisfies respect conflicts and violates composition.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ such that $E \in \mathsf{cf}(F)$ and $E' \notin \mathsf{cf}(F)$. Then $\mathsf{Conflicts}_F(E) = \emptyset$ and $\mathsf{Conflicts}_F(E') \neq \emptyset$, so $E \sqsupset_F^{\mathsf{c\text{-}Conflicts}} E'$ and therefore $E \sqsupset_F^{\mathsf{lex}(\mathsf{c\text{-}Conflicts},\mathsf{Kelly}_{\mathsf{cat}})} E'$, hence respect conflicts is satisfied. $\mathsf{lex}(\sqsupseteq^{\mathsf{c\text{-}Conflicts}}, \sqsupseteq^{\mathsf{Kelly}_{\mathsf{cat}}})$ violates composition.

Consider $F_{13} = (\{a, b, c, d, e\}, \{(a, b), (e, c), (c, d), (c, c), (d, d)\})$ as depicted in Figure 26. $F_{13}$ can be partitioned into two disjoint AFs $F_{13,1} = (\{a, b\}, \{(a, b)\})$ and $F_{13,2} = (\{c, d, e\}, \{(e, c), (c, d), (c, c), (d, d)\})$. The corresponding argument ranking with respect to cat is:

$$e \simeq_{F_{13}}^{\mathsf{cat}} a \succ_{F_{13}}^{\mathsf{cat}} b \succ_{F_{13}}^{\mathsf{cat}} c \succ_{F_{13}}^{\mathsf{cat}} d.$$

Consider sets $\{a, c\}$ and $\{b, d\}$, we get:

$$\{a\} \sqsupset_{F_{13},1}^{\mathsf{lex}(\mathsf{c\text{-}Conflicts},\mathsf{Kelly}_{\mathsf{cat}})} \{b\} \quad \text{and} \quad \{c\} \sqsupset_{F_{13},2}^{\mathsf{lex}(\mathsf{c\text{-}Conflicts},\mathsf{Kelly}_{\mathsf{cat}})} \{d\}.$$

However, since $c \not\succ_{F_{13}}^{\mathsf{cat}} b$, we have:

$$\{a, c\} \asymp_{F_{13}}^{\mathsf{lex}(\mathsf{c\text{-}Conflicts},\mathsf{Kelly}_{\mathsf{cat}})} \{b, d\}.$$

Thus, violating composition.

□

Next, we discuss a variant of the least-discriminating extension-ranking semantics.

Lemma A.13. $\mathsf{lex}(\sqsupseteq^{\mathsf{Conflicts}}, \subseteq)$ *satisfies respect conflicts and violates weak reinstatement.*

Proof. Let $F = (A, R)$ be an AF.

Consider $E, E' \subseteq A$ such that $E \in \mathsf{cf}(F)$ and $E' \notin \mathsf{cf}(F)$. Then $\mathsf{Conflicts}_F(E) = \emptyset$ and $\mathsf{Conflicts}_F(E') \neq \emptyset$, implying:

$$E \sqsupset_F^{\mathsf{Conflicts}} E'.$$

Therefore:

$$E \sqsupset_F^{\mathsf{lex}(\mathsf{Conflicts},\subseteq)} E'.$$

Thus, respect conflicts is satisfied.

Consider $E \subseteq A$ and $a \in A$ such that $a \notin E_F^+ \cup E_F^-$ with $a \in \mathfrak{F}_F(E)$. We know $\mathsf{Conflicts}_F(E) = \mathsf{Conflicts}_F(E \cup \{a\})$, therefore:

$$E \equiv_F^{\mathsf{Conflicts}} E \cup \{a\}.$$

However, $E$ is a subset of $E \cup \{a\}$, meaning:

$$E \sqsupset_F^{\mathsf{lex}(\sqsupseteq^{\mathsf{Conflicts}},\subseteq)} E \cup \{a\}.$$

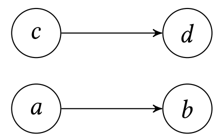


Fig. 27. AF $F_{10}$ from Example 48.

Therefore, weak reinstatement is violated. □

Definition 44. *Let $F = (A, R)$ be an AF and $a, b \in A$ be two specific arguments. We define the* exclusive-or extension-ranking semantics $\mathsf{XOR}_{a,b}$ *as follows:*

$$\text{If } a, b \in E \text{ or } a, b \in E' \text{ then } E \preceq_F^{\mathsf{XOR}_{a,b}} E';$$
$$\text{Otherwise, } E \equiv_F^{\mathsf{XOR}_{a,b}} E'.$$

Lemma A.14. $\mathsf{XOR}_{a,b}$ *satisfies decomposition for any two arguments $a, b$ and violates composition.*

Proof. Let $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$ be an AF and two arguments $a, b \in A_1 \cup A_2$. Assume $E \sqsupseteq_F^{\mathsf{XOR}_{a,b}} E'$. This means that $a, b \notin E$ and $a, b \notin E'$. This also entails, $a, b \notin E \cap A_1$, $a, b \notin E \cap A_2$, $a, b \notin E' \cap A_1$, and $a, b \notin E' \cap A_2$. Hence:

$$E \cap A_1 \sqsupseteq_{F_1}^{\mathsf{XOR}_{a,b}} E' \cap A_1 \quad \text{and} \quad E \cap A_2 \sqsupseteq_{F_1}^{\mathsf{XOR}_{a,b}} E' \cap A_2.$$

Showing that decomposition is satisfied.

Consider $F_{10}$ from Example 48 recalled in Figure 27. For $\mathsf{XOR}_{a,c}$ and sets $\{a, c\}$ and $\{b, d\}$ we get:

$$\{a\} \equiv_{F_{10,1}}^{\mathsf{XOR}_{a,c}} \{b\} \quad \text{and} \quad \{c\} \equiv_{F_{10,2}}^{\mathsf{XOR}_{a,c}} \{d\}.$$

However, we get:

$$\{a, c\} \prec_{F_{10}}^{\mathsf{XOR}_{a,c}} \{b, d\}.$$

Thus, composition is violated. A similar example can be found for any two arguments. □

Definition 45. *Let $F = (A, F)$ be an AF, $\rho$ an argument-ranking semantics and $E, E' \subseteq A$. We define* Elitist extension-ranking semantics $\mathsf{Eli}_\rho$ *via:*

(1) *If $E = E'$ then $E \equiv_F^{\mathsf{Eli}_\rho} E'$;*
(2) *If $E = \emptyset$ and $E' \neq \emptyset$ then $E \sqsupset_F^{\mathsf{Eli}_\rho} E'$;*
(3) *Otherwise $E \sqsupseteq_F^{\mathsf{Eli}_\rho} E'$ if and only if $\exists b \in E'$ s.t. $\forall a \in E$ we have $a \succ_F^\rho b$.*

Lemma A.15. $\mathsf{Eli}_{Cat}$ *satisfies composition and violates weak reinstatement.*

Proof. Let $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$ and argument-ranking semantics $Cat$. Assume $E, E' \subseteq A_1 \cup A_2$ with $E \cap A_1 \sqsupseteq_{F_1}^{\mathsf{Eli}_{Cat}} E' \cap A_1$ and $E \cap A_2 \sqsupseteq_{F_2}^{\mathsf{Eli}_{Cat}} E' \cap A_2$. Then there is an argument $b_1 \in E' \cap A_1$ such that for all $a_1 \in E \cap A_1$, $a_1 \succ_{F_1}^{Cat} b_1$ and for $F_2$, there is an argument $b_2 \in E' \cap A_2$ such that for all $a_2 \in E \cap A_2$, $a_2 \succ_{F_2}^{Cat} b_2$. W.l.o.g. assume $b_1 \succ_F^{Cat} b_2$, then, $a_2 \succ_F^{Cat} b_2$ for all $a_2 \in E \cap A_2$. For all $a_1 \in E \cap A_1$, $a_1 \succ_F b_1$ and because of transitivity, we have, $a_1 \succ_F b_2$. So, there is an argument $b \in E'$ such that for all arguments $a \in E$, $a \succ_F^\rho b$ and therefore $E \sqsupset_F^{\mathsf{Eli}_{Cat}} E'$.

Consider $F_3$ from Example 10 and the h-categoriser argument-ranking semantics. The set $\{h\}$ is the only most plausible non-empty set for $\sqsupseteq_{F_3}^{\mathsf{Eli}_{Cat}}$, every other set is ranked below it, therefore weak reinstatement is violated by $\mathsf{Eli}_{Cat}$. □

Definition 46. *Let* $F = (A, R)$ *be an AF and* $a, b \in A$. *We define the* cardinality-based *argument-ranking semantics* Cb *as follows:*

$$a \succeq_F^{\mathrm{Cb}} b \text{ if and only if } |a_F^-| \leq |b_F^-|.$$

Definition 47. *Let* $F = (A, R)$ *be an AF and* $\rho$ *be an argument-ranking semantics for AF. The* numerical strength value $sv_F^\rho(a) : A \to \mathbb{N}$ *of argument* $a \in A$ *in* $F$ *with respect to* $\rho$ *is defined as the length of the longest sequence of arguments* $a_1, \ldots, a_i$ *such that* $a_1 \succ_F^\rho \cdots \succ_F^\rho a_i \succ_F^\rho a$, *then* $sv_F^\rho(a) = i$. *If there is no* $b \in A$ *such that* $b \succ_F^\rho a$, *then* $sv_F^\rho(a) = 0$.

Definition 48. *Let* $F = (A, R)$ *be an AF,* $E, E' \subseteq A$, *and* Cb *cardinality-based argument-ranking semantics. We define the* order-based extension-ranking semantics $\mathrm{OBE}_{\mathrm{Cb},sum}$ *as follows:*

$$E \sqsupseteq_F^{\mathrm{OBE}_{\mathrm{Cb},sum}} E' \text{ iff } \sum_{a \in E} sv_F^{\mathrm{Cb}}(a) \leq \sum_{b \in E'} sv_F^{\mathrm{Cb}}(b).$$

Lemma A.16. $\mathrm{OBE}_{\mathrm{Cb},sum}$ *satisfies addition robustness.* $\mathrm{OBE}_{\mathrm{Cb},sum}$ *violates weak reinstatement and composition.*

Proof. Let $F = (A, R)$ be an AF, $\succeq_F^{\mathrm{Cb}}$ the argument ranking with respect to Cb, and $E, E' \subseteq A$ such that $E \sqsupseteq_F^{\mathrm{OBE}_{\mathrm{Cb},sum}} E'$. Consider AF $F' = (A, R \cup \{(a, b)\}$ such that $a \in E$, $b \notin E$, and $b \in E'$. For every argument $c \in A \setminus \{b\}$, $c_F^- = c_{F'}^-$, and for $b$, $b_{F'}^- = b_F^- \cup \{a\}$.

- **If** $b \succ_F^{\mathrm{Cb}} c$ then $|b_F^-| < |c_F^-|$ and therefore we have either $b \succ_{F'}^{\mathrm{Cb}} c$ or $b \simeq_{F'}^{\mathrm{Cb}} c$. The first case implies that $sv_F^{\mathrm{Cb}}(b) = sv_{F'}^{\mathrm{Cb}}(b)$ and the second case implies $sv_F^{\mathrm{Cb}}(b) < sv_{F'}^{\mathrm{Cb}}(b)$.
- **If** $b \simeq_F^{\mathrm{Cb}} c$ then $c \succ_{F'}^{\mathrm{Cb}} b$ and this implies $sv_F^{\mathrm{Cb}}(b) < sv_{F'}^{\mathrm{Cb}}(b)$ and $sv_F^{\mathrm{Cb}}(c) > sv_{F'}^{\mathrm{Cb}}(c)$.

Theses two cases show that for $E$, $\sum_{c \in E} sv_F^{\mathrm{Cb}}(c) \geq \sum_{c \in E} sv_{F'}^{\mathrm{Cb}}(c)$. Every time the value $sv_{F'}^{\mathrm{Cb}}(c)$ is lower than in $F$, $sv_{F'}^{\mathrm{Cb}}(b)$ increases and ensuring $\sum_{d \in E'} sv_F^{\mathrm{Cb}}(d) \leq \sum_{d \in E'} sv_{F'}^{\mathrm{Cb}}(d)$. Therefore:

$$E \sqsupseteq_{F'}^{\mathrm{OBE}_{\mathrm{Cb},sum}} E'.$$

□

Example 51. *Consider AF* $F_3$ *from Example 10 and set* $\{h\}$. *Then argument* $j$ *is defended by* $\{h\}$, *but* $j$ *has one attacker implying:*

$$\{h\} \sqsupset_{F_3}^{\mathrm{OBE}_{\mathrm{Cb},sum}} \{h, j\}.$$

*Thus, weak reinstatement is violated.*

Example 52. *Let* $F_{14} = (\{a, b, c, d, e\}, \{(a, b), (b, a), (a, c), (b, c), (c, b), (c, c)\})$ *be an AF as depicted in Figure 28. We get the following argument ranking with respect to* Cb*:*

$$d \equiv_{F_{14}}^{\mathrm{Cb}} e \succ_{F_{14}}^{\mathrm{Cb}} a \succ_{F_{14}}^{\mathrm{Cb}} b \succ_{F_{14}}^{\mathrm{Cb}} c.$$

*Since* $d$ *and* $e$ *have no attackers, they are the highest-ranked argument, followed by* $a$, *which has one attacker. Argument* $b$ *has two attackers, and* $c$ *has three attackers.* $F_{14}$ *can be split into two AFs:* $F_{14,1} = (\{a, b, c\}, \{(a, b), (b, a), (a, c), (b, c), (c, b), (c, c)\})$ *and* $F_{14,2} = (\{d, e\}, \emptyset)$. *For* $F_{14,1}$, *the ranking is:*

$$a \succ_{F_{14,1}}^{\mathrm{Cb}} b \succ_{F_{14,1}}^{\mathrm{Cb}} c.$$

*For* $F_{14,2}$, *we have:*

$$d \simeq_{F_{14,2}}^{\mathrm{Cb}} e.$$

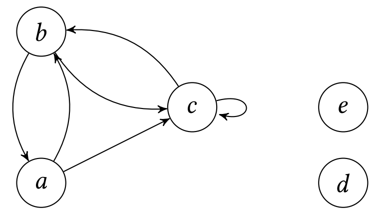


Fig. 28. AF $F_{14}$ for Example 52.

*The corresponding numerical strength values are:*

$$sv^{\mathrm{Cb}}_{F_{14}}(a) = 2, \quad sv^{\mathrm{Cb}}_{F_{14}}(b) = 3, \quad sv^{\mathrm{Cb}}_{F_{14}}(c) = 4,$$
$$sv^{\mathrm{Cb}}_{F_{14}}(d) = 0, \quad sv^{\mathrm{Cb}}_{F_{14}}(e) = 0.$$

$$sv^{\mathrm{Cb}}_{F_{14,1}}(a) = 0, \quad sv^{\mathrm{Cb}}_{F_{14,1}}(b) = 1, \quad sv^{\mathrm{Cb}}_{F_{14,1}}(c) = 2,$$
$$sv^{\mathrm{Cb}}_{F_{14,2}}(d) = 0, \quad sv^{\mathrm{Cb}}_{F_{14,2}}(e) = 0.$$

*Next, consider the sets* $\{a, b, d\}$ *and* $\{c, d\}$*. We have:*

$$\{a, b\} \sqsupset^{\mathrm{OBE}_{\mathrm{Cb},sum}}_{F_{14,1}} \{c\} \quad \textit{and} \quad \{d\} \equiv^{\mathrm{OBE}_{\mathrm{Cb},sum}}_{F_{14,2}} \{d\}.$$

*However, we get:*

$$\{c, d\} \sqsupset^{\mathrm{OBE}_{\mathrm{Cb},sum}}_{F_{14}} \{a, b, d\}.$$

*Therefore, composition is violated by* $\mathrm{OBE}_{\mathrm{Cb},sum}$*.*

Definition 49. *Let* $F = (A, R)$ *be an AF and* $E, E' \subseteq A$*. The* incomparable *extension-ranking semantics* $\sqsupseteq^{\text{incomparable}}$ *is defined via:*

$$E \equiv^{\text{incomparable}}_F E' \text{ if and only if } E = E';$$
$$E \asymp^{\text{incomparable}}_F E' \text{ otherwise.}$$

Lemma A.17. $\sqsupseteq^{\text{incomparable}}$ *satisfies decomposition and violates weak reinstatement.*

Proof. Let $F = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$ be an AF and $E, E' \subseteq A_1 \cup A_2$. By definition, if:

$$E \equiv^{\text{incomparable}}_F E'.$$

Then:

$$E \cap A_1 \equiv^{\text{incomparable}}_{F_1} E' \cap A_1 \quad \text{and} \quad E \cap A_2 \equiv^{\text{incomparable}}_{F_2} E' \cap A_2.$$

Similarly, if:

$$E \asymp^{\text{incomparable}}_F E'.$$

Then:

$$E \cap A_1 \asymp^{\text{incomparable}}_{F_1} E' \cap A_1 \quad \text{and} \quad E \cap A_2 \asymp^{\text{incomparable}}_{F_2} E' \cap A_2.$$

Next assume $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^- \cup E^+)$. Then $E \neq E \cup \{a\}$, hence:

$$E \asymp^{\text{incomparable}}_F E \cup \{a\}.$$

Thus, weak reinstatement is violated. □

**Proposition 5.** *The following holds:*

(1) *The principle respect conflicts implies* cf*-soundness.*
(2) *The principle strong reinstatement implies weak reinstatement.*
(3) *The principles* ad*-generalisation and strong reinstatement are incompatible.*
(4) *The principles $\sigma$-generalisation for $\sigma \in \{\text{co}, \text{pr}, \text{sst}\}$ and addition robustness are incompatible.*
(5) *For the remaining pairs of principles $p_1, p_2 \in \{\sigma$-generalisation, respect conflicts, composition, decomposition, weak reinstatement, strong reinstatement, addition robustness$\}$ where $p_1 \neq p_2$, $p_1$ and $p_2$ are pairwise independent.*

Proof. **to 1** Proposition 2 shows that respect conflicts implies cf-soundness.

**to 2** Let $\tau$ be an extension-ranking semantics that satisfies strong reinstatement. For any $F = (A, R)$ with $E \subseteq A$, if $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E_F^- \cup E_F^+)$, it follows that $E \cup \{a\} \sqsupset_F^\tau E$. Therefore, $E \cup \{a\} \sqsupseteq_F^\tau E$ also holds.

**to 3** Example 16 shows that ad-generalisation and strong reinstatement are incompatible.

**to 4** Example 50 shows that $\sigma$-generalisation for $\sigma \in \{\text{co}, \text{pr}, \text{sst}\}$ and addition robustness are incompatible.

**to 5** To show the pairwise independence of the remaining pairs of principles, for every pair $p_1$ and $p_2$, we present:
- One extension-ranking semantics that satisfies $p_1$ and $p_2$;
- One extension-ranking semantics that satisfies $p_1$ but violates $p_2$;
- One extension-ranking semantics that violates $p_1$ but satisfies $p_2$.

**($\sigma$-generalisation, respect conflicts):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\text{LD}^\sigma$ satisfies $\sigma$-generalisation and violates respect conflicts (see Table 1);
- $\sqsupseteq^{\text{Conflicts}}$ violates $\sigma$-generalisation and satisfies respect conflicts (see Table 2).

**($\sigma$-generalisation, composition):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ ($\sigma = \text{ad}$) satisfies $\sigma$-generalisation and violates composition (see Table 6);
- $\sqsupseteq^{\text{Conflicts}}$ violates $\sigma$-generalisation and satisfies composition (see Table 2).

**($\sigma$-generalisation, decomposition):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\text{LD}^\sigma$ satisfies $\sigma$-generalisation and violates decomposition (see Table 1);
- $\sqsupseteq^{\text{Conflicts}}$ violates $\sigma$-generalisation and satisfies decomposition (see Table 2).

**($\sigma$-generalisation, weak reinstatement):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\text{LD}^{\sigma*}$ satisfies $\sigma$-generalisation and violates weak reinstatement (see Lemma A.11);
- $\sqsupseteq^{\text{Conflicts}}$ violates $\sigma$-generalisation and satisfies weak reinstatement (see Table 2).

**($\sigma$-generalisation, strong reinstatement):**
- r-$\sigma$ ($\sigma \in \{\text{co}, \text{gr}, \text{pr}, \text{sst}\}$) satisfies both principles (see Table 4);
- $\text{LD}^\sigma$ satisfies $\sigma$-generalisation and violates strong reinstatement (see Table 1);
- $\sqsupseteq^{\text{Condef}}$ violates $\sigma$-generalisation and satisfies strong reinstatement (see Table 2).

**(ad-generalisation, addition robustness):**
- $\text{LD}^{\text{ad}}$ satisfies both principles (see Table 1);
- $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ satisfies ad-generalisation and violates addition robustness (see Table 6);
- $\sqsupseteq^{\text{Conflicts}}$ violates ad-generalisation and satisfies addition robustness (see Table 2).

**(respect conflicts, composition):**
- r-$\sigma$ satisfies both principles (see Table 4);

- $\text{lex}(\sqsupseteq^{\text{c-Conflicts}}, \sqsupseteq^{\text{Kelly}_{\text{cat}}})$ satisfies respect conflicts and violates composition (see Lemma A.12);
- $\sqsupseteq^{\text{Undef}}$ violates respect conflicts and satisfies composition (see Table 2).

**(respect conflicts, decomposition):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\text{LD}^{\text{Conflicts}}$ satisfies respect conflicts and violates decomposition (see Table 1);
- $\sqsupseteq^{\text{Undef}}$ violates respect conflicts and satisfies decomposition (see Table 2).

**(respect conflicts, weak reinstatement):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\text{lex}(\sqsupseteq^{\text{Conflicts}}, \subseteq)$ satisfies respect conflicts and violates weak reinstatement (see Lemma A.13);
- $\sqsupseteq^{\text{Undef}}$ violates respect conflicts and satisfies weak reinstatement (see Table 2).

**(respect conflicts, strong reinstatement):**
- r-co satisfies both principles (see Table 4);
- $\sqsupseteq^{\text{Conflicts}}$ satisfies respect conflicts and violates strong reinstatement (see Table 2);
- $\sqsupseteq^{\text{Condef}}$ violates respect conflicts and satisfies strong reinstatement (see Table 2).

**(respect conflicts, addition robustness):**
- r-ad satisfies both principles (see Table 4);
- r-co satisfies respect conflicts and violates addition robustness (see Table 4);
- $\sqsupseteq^{\text{Unatt}}$ violates respect conflicts and satisfies addition robustness (see Table 2).

**(composition, decomposition):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\sqsupseteq^{\text{Conflicts}}$ satisfies composition and violates decomposition (see Table 2);
- $\text{XOR}_{a,b}$ violates composition and satisfies decomposition (see Lemma A.14).

**(composition, weak reinstatement):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\text{Eli}_{Cat}$ satisfies composition and violates weak reinstatement (see Lemma A.15);
- $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ violates composition and satisfies weak reinstatement (see Table 6).

**(composition, strong reinstatement):**
- r-co satisfies both principles (see Table 4);
- r-ad satisfies composition and violates strong reinstatement (see Table 4);
- $\text{cope}(\sqsupseteq^{\text{nonatt}})$ violates composition and satisfies strong reinstatement (see Table 6).

**(composition, addition robustness):**
- r-ad satisfies both principles (see Table 4);
- r-co satisfies composition and violates addition robustness (see Table 4);
- $\text{OBE}_{sv^{\text{Cb}},sum}$ violates composition and satisfies addition robustness (see Lemma A.16).

**(decomposition, weak reinstatement):**
- r-$\sigma$ satisfies both principles (see Table 4);
- $\sqsupseteq^{\text{incomparable}}$ satisfies decomposition and violates *weak reinstatement* (see Lemma A.17);
- $\text{LD}^{\sigma}$ violates decomposition and satisfies weak reinstatement (see Table 1).

**(decomposition, strong reinstatement):**
- r-co satisfies both principles (see Table 4);
- r-ad satisfies decomposition and violates strong reinstatement (see Table 4);
- $\text{cope}(\sqsupseteq^{\text{nonatt}})$ violates decomposition and satisfies strong reinstatement (see Table 6).

**(decomposition, addition robustness):**
- r-ad satisfies both principles (see Table 4);
- r-co satisfies decomposition and violates addition robustness (see Table 4);

- $\mathrm{LD}^{\mathrm{ad}}$ violates decomposition and satisfies addition robustness (see Table 1).

**(weak reinstatement, strong reinstatement):**
- r-co satisfies both principles (see Table 4);
- r-ad satisfies weak reinstatement and violates strong reinstatement (see Table 4).

**(weak reinstatement, addition robustness):**
- r-ad satisfies both principles (see Table 4);
- r-co satisfies weak reinstatement and violates addition robustness (see Table 4);
- $\mathrm{OBE}_{sv^{\mathrm{Cb}},sum}$ violates weak reinstatement and satisfies addition robustness (see Lemma A.16).

**(strong reinstatement, addition robustness):**
- r-co satisfies strong reinstatement and violates addition robustness (see Table 4);
- $\mathrm{LD}^{\mathrm{ad}}$ violates strong reinstatement and satisfies addition robustness (see Table 1). □

## A.3 Technical Proofs for Section 5

**Proposition 6.** *Let $F = (A, R)$ be an AF and $E \subseteq A$. Then exists a $k \in \mathbb{N}$ with:*

$$E \subseteq \mathfrak{F}^*_{1,F}(E) \subseteq \mathfrak{F}^*_{2,F}(E) \subseteq \dots \subseteq \mathfrak{F}^*_{k-1,F}(E) \subseteq \mathfrak{F}^*_{k,F}(E) = \mathfrak{F}^*_{k+1,F}(E) = \dots$$

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. We show that $\mathfrak{F}^*_{i,F}(E) \subseteq \mathfrak{F}^*_{i+1,F}(E)$.

Observe that $\mathfrak{F}^*_{i,F}(E)$ is formed by adding arguments from $(\mathfrak{F}_F(\mathfrak{F}^*_{i-1,F}(E)) \setminus E^-)$, without removing any arguments:

$$\mathfrak{F}^*_{i,F}(E) = \mathfrak{F}^*_{i-1,F}(E) \cup (\mathfrak{F}_F(\mathfrak{F}^*_{i-1,F}(E)) \setminus E^-).$$

Therefore:

$$\mathfrak{F}^*_{i-1,F}(E) \cup (\mathfrak{F}_F(\mathfrak{F}^*_{i-1,F}(E)) \setminus E^-) \subseteq$$
$$(\mathfrak{F}^*_{i-1,F}(E) \cup (\mathfrak{F}_F(\mathfrak{F}^*_{i-1,F}(E)) \setminus E^-)) \cup (\mathfrak{F}_F(\mathfrak{F}^*_{i-1,F}(E) \cup (\mathfrak{F}_F(\mathfrak{F}^*_{i-1,F}(E)) \setminus E^-)) \setminus E^-).$$

This is equivalent to:

$$\mathfrak{F}^*_{i,F} \subseteq \mathfrak{F}^*_{i+1,F}.$$

Since $\mathfrak{F}^*_{i,F} \subseteq \mathfrak{F}^*_{i+1,F}$ holds for every $i$, there must exist a smallest $k$ such that $\mathfrak{F}^*_{k,F} = \mathfrak{F}^*_{k+1,F}$, because the number of arguments is finite. Therefore, $\mathfrak{F}^*_{k,F} = \mathfrak{F}^*_{k',F}$ for all $k' > k$. □

**Proposition 7.** *Let $F = (A, R)$ be an AF, $E \subseteq A$ such that $E \in \mathrm{ad}(F)$. Then $\mathfrak{F}_F(E) = \mathfrak{F}^*_F(E)$.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$ such that $E \in \mathrm{ad}(F)$. Applying $\mathfrak{F}^*$ to $E$, we get: $\mathfrak{F}^*_{1,F}(E) = E$ and $\mathfrak{F}^*_{2,F}(E) = E \cup (\mathfrak{F}_F(E) \setminus E^-)$. $\mathfrak{F}_F(E)$ is defined such that this function will not return any attacker of $E$ if $E$ is admissible, so we do not need to remove any attacker, thus, $\mathfrak{F}_F(E) \setminus E^- = \mathfrak{F}_F(E)$. The *Fundamental Lemma* (Dung 1995) then implies $E \subseteq \mathfrak{F}_F(E)$ if $E$ is admissible, $E \cup \mathfrak{F}_F(E) = \mathfrak{F}_F(E)$. Therefore, $\mathfrak{F}^*_{2,F}(E) = \mathfrak{F}_F(E)$. We can use the same reasoning for any $i > 2$ and entailing that in every step $\mathfrak{F}^*_F(E)$ return the same arguments as $\mathfrak{F}_F(E)$. □

**Proposition 8.** *Let $F = (A, R)$ be an AF and $E = \mathrm{gr}(F)$, then $E$ is the least fixed point of $\mathfrak{F}^*_F$.*

Proof. Let $F = (A, R)$ be an AF and $E = \mathrm{gr}(F)$. To prove that $E$ is the least fixed point of $\mathfrak{F}^*_F$, we modify the algorithm to calculate the grounded extension (for details see (Baroni, Gabbay, et al. 2018)). This algorithm starts with the empty set, $\mathfrak{F}_F(\emptyset)$, and add in every step all defended arguments into that set until a fixed point is reached. Since $\emptyset \in \mathrm{ad}(F)$, $\mathfrak{F}_F(\emptyset) = \mathfrak{F}^*_F(\emptyset)$ like shown in Proposition 7 and if $\mathfrak{F}_F(\emptyset)$ reaches a fixed point, then $\mathfrak{F}^*_F(\emptyset)$ reaches a fixed point. Since the grounded extension $E$ is an admissible set, $\mathfrak{F}_F(E) = \mathfrak{F}^*_F(E)$. This fixed point of $\mathfrak{F}^*_F(\emptyset)$ coincided with the grounded extension $E$ and therefore $E$ is the least fixed point of $\mathfrak{F}^*_F$. □

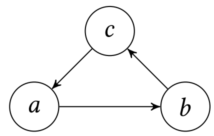


Fig. 29. Recalled AF $F_5$ from Example 15.

**Proposition 9.** *Let $F = (A, R)$ be an AF and $E \in \text{co}(F)$. Then $\mathfrak{F}^*_F(E) = E$.*

Proof. Let $F = (A, R)$ be an AF and $E \in \text{co}(F)$, by definition of the complete semantics, $E \in \text{ad}(F)$ and therefore $\mathfrak{F}_F(E) = \mathfrak{F}^*_F(E)$. Since, $E$ already contains every argument it defends, no more arguments will be added when applying the characteristic function, $\mathfrak{F}_F(E) \subseteq E$. The other direction is shown by Proposition 7. Thus, $\mathfrak{F}^*_F(E) = E$. □

**Proposition 10.** *The base relation $\sqsupseteq^\tau_F$ is reflexive and transitive for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.*

Proof. Consider AF $F = (A, R)$, $E \subseteq A$, and $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.

**Reflexive** Let $\mathcal{E} = \tau_F(E)$ and $E' = E$, then $\tau_F(E') = \mathcal{E}$ and therefore $E \equiv^\tau_F E'$.

**Transitive** Let $E', E'' \subseteq A$ be two sets of arguments such that $E \sqsupseteq^\tau_F E'$ and $E' \sqsupseteq^\tau_F E''$. Then $\tau_F(E) \subseteq \tau_F(E')$ and $\tau_F(E') \subseteq \tau_F(E'')$, therefore $E \sqsupseteq^\tau_F E''$. □

**Proposition 11.** *$\sqsupseteq^\tau$ satisfies and violates the principles as depicted in Table 2 for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$.*

Proof. We consider each principle as follows.

Lemma A.18. *$\sqsupseteq^{\text{Conflicts}}$ violates $\sigma$-generalisation for $\sigma \in \{\text{cf}, \text{ad}, \text{co}, \text{gr}, \text{pr}, \text{sst}\}$.*

Example 53. *Consider AF $F_2$ from Example 3 and the sets $\{a\}$ and $\{b\}$. The set $\{a\}$ is admissible, while $\{b\}$ is not admissible. However, both sets are conflict-free, implying:*

$$\{a\} \equiv^{\text{Conflicts}}_{F_2} \{b\}.$$

*Therefore, $\sigma$-generalisation for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{st}, \text{sst}\}$ is violated by $\sqsupseteq^{\text{Conflicts}}$.*

Lemma A.19. *$\sqsupseteq^\tau$ violates $\sigma$-generalisation and respect conflicts for $\sigma \in \{\text{cf}, \text{ad}, \text{co}, \text{gr}, \text{pr}, \text{sst}\}$ with $\tau \in \{\text{Undef}, \text{Condef}, \text{Unatt}\}$.*

Example 54. *Consider AF $F_5$ from Example 15, recalled in Figure 29. The set $\{a, b, c\}$ defends all its elements and contains all argument, so:*

$$\text{Undef}_{F_5}(\{a, b, c\}) = \text{Condef}_{F_5}(\{a, b, c\}) = \text{Unatt}_{F_5}(\{a, b, c\}) = \emptyset.$$

*Therefore, there cannot be a set strictly more plausible to accept than $\{a, b, c\}$ with respect to $\sqsupseteq^\tau$ for $\tau \in \{\text{Undef}, \text{Condef}, \text{Unatt}\}$, implying $\{a, b, c\} \in \max_{\sqsupseteq^\tau}(F_5)$ and therefore $\sigma$-generalisation and respect conflicts for $\sigma \in \{\text{cf}, \text{ad}, \text{co}, \text{gr}, \text{pr}, \text{sst}\}$ are violated.*

Lemma A.20. *$\sqsupseteq^{\text{Conflicts}}$ satisfies respect conflicts.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$ such that $E \in \text{cf}(F)$. $E$ is conflict-free and therefore there is no pair $a, b \in E$ such that $(a, b) \in R$, thus $\text{Conflicts}_F(E) = \emptyset$ and $E \in \max_{\sqsupseteq^{\text{Conflicts}}}(F)$.

Consider $E' \subseteq A$ such that $E' \notin \text{cf}(F)$, then there is one pair $a, b \in E'$ such that $(a, b) \in R$, thus $\text{Conflicts}_F(E) \neq \emptyset$ and $E' \notin \max_{\sqsupseteq^{\text{Conflicts}}}(F)$. Therefore, respect conflicts is satisfied. □

The following lemma will be useful to prove (de)composition.

Lemma A.21. *For* $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$*, if AFs* $F_1 = (A_1, R_1)$*,* $F_2 = (A_2, R_2)$ *and* $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ *with* $A_1 \cap A_2 = \emptyset$ *then* $\tau_{F_1}(E \cap A_1) \cup \tau_{F_2}(E \cap A_2) = \tau_F(E)$ *for every* $E \subseteq A_1 \cup A_2$.

Proof. We prove it for $\tau = \mathsf{Undef}$ (others are similar). Suppose $F_1 = (A_1, R_1)$, $F_2 = (A_2, R_2)$ and $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$. We prove that $\mathsf{Undef}_{F_1}(E \cap A_1) \cup \mathsf{Undef}_{F_2}(E \cap A_2) = \mathsf{Undef}_F(E)$.

- **"$\subseteq$":** Suppose w.l.o.g. that argument $a \in \mathsf{Undef}_{F_1}(E \cap A_1)$. Then $a \in E \cap A_1$ and $a \notin \mathfrak{F}_{F_1}(E \cap A_1)$. Therefore, $a \in E$ and since $F_1$ and $F_2$ are disjoint, $a \notin \mathfrak{F}_F(E)$. Implying $a \in \mathsf{Undef}_F(E)$.
- **"$\supseteq$":** Suppose $a \in \mathsf{Undef}_F(E)$. Furthermore, suppose w.l.o.g. that $a \in F_1$. Then $a \in E$ and $a \notin \mathfrak{F}_F(E)$. Hence, there is an $b \in A_1 \cup A_2$ such that $b$ attacks $a$ and $b$ is not attacked by $E$. It then follows that $b \in A_1$. Thus, $a \notin \mathfrak{F}_{F_1}(E \cap A_1)$. Implying, $a \in \mathsf{Undef}_{F_1}(E \cap A_1)$. □

Lemma A.22. $\sqsupseteq_F^\tau$ *satisfies composition and decomposition for* $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$.

Proof. Let $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$, $F_1 = (A_1, R_1)$, $F_2 = (A_2, R_2)$, and $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$.

- **"Composition":** Suppose $E \cap A_1 \sqsupseteq_{F_1}^\tau E' \cap A_1$ and $E \cap A_2 \sqsupseteq_{F_1}^\tau E' \cap A_2$. Then $\tau_{F_1}(E \cap A_1) \subseteq \tau_{F_1}(E' \cap A_1)$ and $\tau_{F_2}(E \cap A_2) \subseteq \tau_{F_2}(E' \cap A_2)$. Lemma A.21 then implies that $\tau_F(E) \subseteq \tau_F(E')$. Thus, $E \sqsupseteq_F^\tau E'$.
- **"Decomposition":** Suppose $E \sqsupseteq_F^\tau E'$. Then $\tau_F(E) \subseteq \tau_F(E')$. Lemma A.21 then implies that $\tau_{F_1}(E \cap A_1) \subseteq \tau_{F_1}(E' \cap A_1)$ and $\tau_{F_2}(E \cap A_2) \subseteq \tau_{F_2}(E' \cap A_2)$. Thus, $E \cap A_1 \sqsupseteq_{F_1}^\tau E' \cap A_1$ and $E \cap A_2 \sqsupseteq_{F_2}^\tau E' \cap A_2$. □

Lemma A.23. $\sqsupseteq^\tau$ *satisfies strong reinstatement for* $\tau \in \{\mathsf{Condef}, \mathsf{Unatt}\}$ *and weak reinstatement for* $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}\}$.

Proof. Let $F = (A, R)$ be an AF, $E \subseteq A$, and $a \in A$ such that $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin E_F^+ \cup E_F^-$.

- **"$\tau = \mathsf{Conflicts}$":** Since $a \notin E_F^+ \cup E_F^-$ and $a \in \mathfrak{F}_F(E)$, $\mathsf{Conflicts}_F(E) = \mathsf{Conflicts}_F(E \cup \{a\})$, thus $E \cup \{a\} \equiv_F^{\mathsf{Conflicts}} E$. Therefore, weak reinstatement is satisfied and strong reinstatement is violated.
- **"$\tau = \mathsf{Undef}$":** Since $a \in \mathfrak{F}_F(E)$, $\mathsf{Undef}_F(E \cup \{a\}) \subseteq \mathsf{Undef}_F(E)$, implying $E \cup \{a\} \sqsupseteq_F^{\mathsf{Undef}} E$. However, if $a_F^+ = \emptyset$, then $\mathsf{Undef}_F(E \cup \{a\}) = \mathsf{Undef}_F(E)$ and therefore $E \cup \{a\} \equiv_F^{\mathsf{Undef}} E$. Therefore, weak reinstatement is satisfied and strong reinstatement is violated.
- **"$\tau = \mathsf{Condef}$":** Since $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin E_F^+ \cup E_F^-$, $a \in \mathsf{Condef}_F(E)$. We have $\mathfrak{F}_{2,F}^*(E) = E \cup (\mathfrak{F}_F(E) \setminus E_F^-)$ and $a \in (\mathfrak{F}_F(E) \setminus E_F^-)$, implying $\mathfrak{F}_{2,F}^*(E) \subseteq \mathfrak{F}_{2,F}^*(E \cup \{a\})$. Then Proposition 6 entails $\mathfrak{F}_F^*(E) = \mathfrak{F}_F^*(E \cup \{a\})$, thus $\mathsf{Condef}(E \cup \{a\}) \subset \mathsf{Condef}(E)$ and $E \cup \{a\} \sqsupset_F^{\mathsf{Condef}} E$. Therefore, strong reinstatement is satisfied.
- **"$\tau = \mathsf{Unatt}$":** Since $a \notin E \cup E_F^+$, $a \in \mathsf{Unatt}(E)$ and adding $a$ does not remove any argument from $E_F^+$, so $E_F^+ \subseteq (E \cup \{a\})_F^+$. Thus, $\mathsf{Unatt}(E \cup \{a\}) \subset \mathsf{Unatt}(E)$ and $E \cup \{a\} \sqsupset_F^{\mathsf{Unatt}} E$. Therefore, strong reinstatement is satisfied. □

Lemma A.24. $\sqsupseteq^\tau$ *satisfies addition robustness for* $\tau \in \{\mathsf{Conflicts}, \mathsf{Unatt}\}$.

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ such that $E \sqsupseteq_F^\tau E'$ with respect to $\tau \in \{\mathsf{Conflicts}, \mathsf{Unatt}\}$. Suppose AF $F' = (A, R \cup \{(a, b)\}$ such that $a \in E$, $b \notin E$, and $b \in E'$.

- **"$\tau = \mathsf{Conflicts}$":** Since $b \notin E$, $\mathsf{Conflicts}_F(E) = \mathsf{Conflicts}_{F'}(E)$ and $\mathsf{Conflicts}_F(E') \subseteq \mathsf{Conflicts}_{F'}(E')$, if $E \sqsupseteq_F^{\mathsf{Conflicts}} E'$ then $E \sqsupseteq_{F'}^{\mathsf{Conflicts}} E'$.
- **"$\tau = \mathsf{Unatt}$":** For every attack $r$ in $R$, $r \in R \cup \{(a, b)\}$. Hence, for every argument $c \in A$, $c \neq a$, $c_F^+ = c_{F'}^+$. Thus, $\mathsf{Unatt}_F(E) \supseteq \mathsf{Unatt}_{F'}(E)$ and $\mathsf{Unatt}_F(E') = \mathsf{Unatt}_{F'}(E')$, therefore if $E \sqsupseteq_F^{\mathsf{Unatt}} E'$ then $E \sqsupseteq_{F'}^{\mathsf{Unatt}} E'$. □

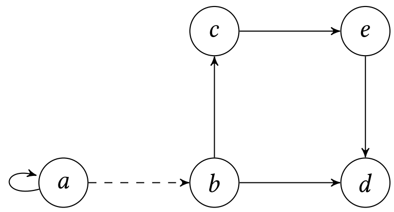


Fig. 30. Recalled AF $F_7$ from Example 31, where the attack $(a, b)$ is added later to obtain $F'_7$.

Lemma A.25. *$\sqsupseteq^{\tau}$ violates addition robustness for $\tau \in \{\mathsf{Undef}, \mathsf{Condef}\}$.*

Example 55. *Consider AF $F_7 = (\{a, b, c, d, e\}, \{(a, a), (b, c), (b, d), (c, e), (e, d)\})$ as recalled in Figure 30 and the sets $\{a, d\}$ and $\{a, b, c, d\}$. Argument $d$ is undefended by both sets, and additionally $\{a, b, c, d\}$ does not defend argument $c$, so $\mathsf{Undef}_{F_7}(\{a, d\}) = \{d\}$ and $\mathsf{Undef}_{F_7}(\{a, b, c, d\}) = \{c, d\}$ and therefore:*

$$\{a, d\} \sqsupset^{\mathsf{Undef}}_{F_7} \{a, b, c, d\}.$$

*We add the attack $(a, b)$ into $F_7$ to obtain $F'_7$. However, in $F'_7$ the set $\{a, b, c, d\}$ defends all its elements, i. e., $\mathsf{Undef}_{F'_7}(\{a, b, c, d\}) = \emptyset$, while argument $d$ is still not defended by $\{a, d\}$. This implies:*

$$\{a, b, c, d\} \sqsupset^{\mathsf{Undef}}_{F'_7} \{a, d\}.$$

*Therefore $\sqsupseteq^{\mathsf{Undef}}$ violates addition robustness.*

*For $\sqsupseteq^{\mathsf{Condef}}$, we get $\mathsf{Condef}_{F_7}(\{a, d\}) = \mathsf{Condef}_{F_7}(\{a, b, c, d\}) = \emptyset$. We add the attack $(a, b)$ and create $F'_7$. However, in $F'_7$, argument $c$ is consistently defended by $\{a, d\}$, hence $\mathsf{Condef}_{F'_7}(\{a, d\}) = \{c\}$. For set $\{a, b, c, d\}$, $\mathsf{Condef}_{F'_7}(\{a, b, c, d\}) = \emptyset$, implying:*

$$\{a, b, c, d\} \sqsupset^{\mathsf{Condef}}_{F'_7} \{a, d\}.$$

*Therefore addition robustness is violated.*

Lemma A.26. *$\sqsupseteq^{\tau}$ satisfies syntax independence for $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$.*

Proof. Clear by definition. □

□

**Proposition 12.** *For AF $F = (A, R)$, sets $E, E' \subseteq A$, and $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$, it holds that if $E \sqsupseteq^{\tau}_F E'$ then $E \sqsupseteq^{\text{c-}\tau}_F E'$.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$. Consider base relations $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$ and $E \sqsupseteq^{\tau}_F E'$. Then $\tau_F(E) \subseteq \tau_F(E')$, this also implies $|\tau_F(E)| \leq |\tau_F(E')|$, thus $E \sqsupseteq^{\text{c-}\tau}_F E'$. □

**Proposition 13.** *The base relation $\sqsupseteq^{\text{c-}\tau}_F$ is reflexive and transitive for $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$.*

Proof. Consider AF $F = (A, R)$, $E \subseteq A$, and $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$.

**Reflexive** Let $\mathcal{E} = \text{c-}\tau_F(E)$ and $E' = E$, then $\text{c-}\tau_F(E') = \mathcal{E}$ and therefore $E \equiv^{\text{c-}\tau}_F E'$.

**Transitive** Let $E', E'' \subseteq A$ be two sets of arguments such that $E \sqsupseteq^{\tau}_F E'$ and $E' \sqsupseteq^{\tau}_F E''$. Then $\text{c-}\tau_F(E) \leq \text{c-}\tau_F(E')$ and $\text{c-}\tau_F(E') \leq \text{c-}\tau_F(E'')$, therefore $E \sqsupseteq^{\text{c-}\tau}_F E''$. □

**Proposition 14.** *$\sqsupseteq_F^{\text{c-}\tau}$ satisfies and violates the principles as depicted in Table 2 for $\tau \in$ {Conflicts, Undef, Condef, Unatt}.*

Proof. We consider each principle as follows.

Lemma A.27. *The following holds:*

(1) *$\sqsupseteq^{\text{c-}\tau}$ violates $\sigma$-generalisation for $\sigma \in$ {ad, co, gr, pr, st, sst} with $\tau \in$ {Conflicts, Undef, Condef, Unatt}.*
(2) *$\sqsupseteq^{\text{c-}\tau}$ violates respect conflicts for $\tau \in$ {Undef, Condef, Unatt}.*
(3) *$\sqsupseteq^{\text{c-}\tau}$ violates addition robustness for $\tau \in$ {Undef, Condef}.*

Proof. Proposition 12 shows that $\sqsupseteq^{\text{c-}\tau}$ is a generalisation of $\sqsupseteq^{\tau}$, thus we can use the same counterexamples as for $\sqsupseteq^{\tau}$. So, for (1) and (2) we use Example 54, for (3) we use Example 31. □

Lemma A.28. *$\sqsupseteq^{\text{c-Conflicts}}$ satisfies respect conflicts.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \in A$ such that $E \in \text{cf}(F)$ and $E' \notin \text{cf}(F)$. Since $E \in \text{cf}(F)$, $\text{Conflicts}_F(E) = \emptyset$ and therefore $|\text{Conflicts}_F(E)| = 0$. For $E'$, $\text{Conflicts}_F(E') \neq \emptyset$, hence $|\text{Conflicts}_F(E')| > 0$ and $E \sqsupset_F^{\text{c-Conflicts}} E'$. □

To show the satisfaction of composition, we show a similar lemma to Lemma A.21.

Lemma A.29. *For $\tau \in$ {Conflicts, Undef, Condef, Unatt}, if $F_1 = (A_1, R_1)$, $F_2 = (A_2, R_2)$ and $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$ then $|\tau_{F_1}(E \cap A_1)| + |\tau_{F_2}(E \cap A_2)| = |\tau_F(E)|$ for every $E \subseteq A_1 \cup A_2$.*

Proof. Suppose $F_1 = (A_1, R_1)$, $F_2 = (A_2, R_2)$ and $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$. Because of Lemma A.21, $\tau_{F_1}(E \cap A_1) \cup \tau_{F_2}(E \cap A_2) = \tau_F(E)$. Every element in $\tau_{F_1}(E \cap A_1) \cup \tau_{F_2}(E \cap A_2)$ is in $\tau_F(E)$ and the other way around. Since $A_1 \cap A_2 = \emptyset$, $\tau_{F_1}(E \cap A_1)$ and $\tau_{F_2}(E \cap A_2)$ are disjoint and therefore every element of $\tau_F(E)$ can only be in $\tau_{F_1}(E \cap A_1)$ or $\tau_{F_2}(E \cap A_2)$. Thus, $|\text{Conflicts}_F(E')| > 0$ and $E \sqsupset_F^{\text{c-Conflicts}} E'$. □

Lemma A.30. *$\sqsupseteq^{\text{c-}\tau}$ satisfies composition for $\tau \in$ {Conflicts, Undef, Condef, Unatt}.*

Proof. Let $\tau \in$ {Conflicts, Undef, Condef, Unatt}, $F_1 = (A_1, R_1)$, $F_2 = (A_2, R_2)$ and $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$.

Suppose $E \cap A_1 \sqsupseteq_{F_1}^{\text{c-}\tau} E' \cap A_1$ and $E \cap A_2 \sqsupseteq_{F_2}^{\text{c-}\tau} E' \cap A_2$. Then $|\tau_{F_1}(E \cap A_1)| \leq |\tau_{F_1}(E' \cap A_1)|$ and $|\tau_{F_2}(E \cap A_2)| \leq |\tau_{F_2}(E' \cap A_2)|$. Lemma A.29 then implies $|\tau_F(E)| \leq |\tau_F(E')|$ and therefore $E \sqsupseteq_F^{\text{c-}\tau} E'$. □

Lemma A.31. *$\sqsupseteq^{\text{c-}\tau}$ violates decomposition for $\tau \in$ {Conflicts, Undef, Condef, Unatt}.*

Example 56. *Consider AF $F_{15} = (\{a, b, c, d, e, f, g, h\}, \{(a, b), (a, c), (b, d), (c, e), (f, g), (g, h)\})$ as depicted in Figure 31 and $\tau \in$ {Conflicts, Undef, Condef, Unatt}. This AF can be partitioned into two disjoint AFs $F_{15,1} = (\{a, b, c, d, e\}, \{(a, b), (a, c), (b, d), (c, e)\})$ and $F_{15,2} = (\{f, g, h\}, \{(f, g), (g, h)\})$. Suppose sets $\{a, b, c\}$ and $\{f, g\}$, then $|\tau_{F_{15}}(\{a, b, c\})| = 2$ and $|\tau_{F_{15}}(\{f, g\})| = 1$. However, we also get:*

$$|\text{Conflicts}_{F_{15,1}}(\{a, b, c\} \cap \{a, b, c, d, e\})| = 2, \qquad |\text{Conflicts}_{F_{15,2}}(\{a, b, c\} \cap \{f, g, h\})| = 0,$$
$$|\text{Conflicts}_{F_{15,1}}(\{f, g\} \cap \{a, b, c, d, e\})| = 0, \qquad |\text{Conflicts}_{F_{15,2}}(\{f, g\} \cap \{f, g, h\})| = 1.$$

*Thus, we get:*

$$\{f, g\} \sqsupset_{F_{15}}^{\text{c-Conflicts}} \{a, b, c\} \quad \text{but} \quad \{a, b, c\} \cap \{f, g, h\} \sqsupset_{F_{15,2}}^{\text{c-Conflicts}} \{f, g\}.$$

*Next, consider the sets $\{g\}$ and $\{b, c\}$. For these two sets, we get:*

$$\{g\} \sqsupset_{F_{15}}^{\text{c-Undef}} \{b, c\} \quad \text{but} \quad \{b, c\} \cap \{f, g, h\} \sqsupset_{F_{15,2}}^{\text{c-Undef}} \{g\} \cap \{f, g, h\}.$$

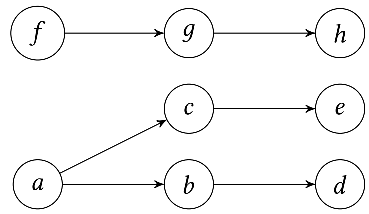


Fig. 31. AF $F_{15}$ from Example 56.

*For sets $\{f\}$ and $\{a\}$, we get:*

$$\{f\} \equiv^{\text{c-Condef}}_{F_{15}} \{a\},\ \textit{but}$$
$$\{a\} \cap \{a,b,c,d,e\} \sqsupset^{\text{c-Undef}}_{F_{15},1} \{f\} \cap \{a,b,c,d,e\},\ \textit{and}$$
$$\{f\} \cap \{f,g,h\} \sqsupset^{\text{c-Undef}}_{F_{15},2} \{a\} \cap \{f,g,h\}.$$

*Consider the sets $\{f\}$ and $\{a\}$, we have:*

$$\{a\} \sqsupset^{\text{c-Unatt}}_{F_{15}} \{f\}\ \textit{and}\ \{f\} \cap \{f,g,h\} \sqsupset^{\text{c-Unatt}}_{F_{15},2} \{a\} \cap \{f,g,h\}.$$

*Thus, for $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$, $\sqsupseteq^{\text{c-}\tau}$ violates decomposition.*

Lemma A.32. *$\sqsupseteq^{\text{c-}\tau}$ satisfies strong reinstatement for $\tau \in \{\text{Condef}, \text{Unatt}\}$ and weak reinstatement for $\tau \in \{\text{Conflicts}, \text{Undef}\}$.*

Proof. Let $F = (A, R)$ be an AF, $E \subseteq A$ and $a \in A$ such that $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin E^+_F \cup E^-_F$.

- **"$\tau$ = Conflicts":** Since $a \notin E^+_F \cup E^-_F$ and $a \in \mathfrak{F}_F(E)$, $|\text{Conflicts}_F(E)| = |\text{Conflicts}_F(E \cup \{a\})|$, thus $E \cup \{a\} \equiv^{\text{c-Conflicts}}_F E$. Therefore, weak reinstatement is satisfied and strong reinstatement is violated.
- **"$\tau$ = Undef":** Since $a \in \mathfrak{F}_F(E)$, $|\text{Undef}_F(E \cup \{a\})| \leq |\text{Undef}_F(E)|$, implying $E \cup \{a\} \sqsupseteq^{\text{c-Undef}}_F E$. However, if $a^+_F = \emptyset$, then $|\text{Undef}_F(E \cup \{a\})| = |\text{Undef}_F(E)|$ and $E \cup \{a\} \equiv^{\text{c-Undef}}_F E$. Therefore, weak reinstatement is satisfied and strong reinstatement is violated.
- **"$\tau$ = Condef":** Since $a \in \mathfrak{F}_F(E)$ and $a \notin E \cup E^+_F \cup E^-_F$, $a \in \text{Condef}_F(E)$. We have $\mathfrak{F}^*_{2,F}(E) = E \cup (\mathfrak{F}_F(E) \setminus E^-_F)$ and $a \in (\mathfrak{F}_F(E) \setminus E^-_F)$, implying $\mathfrak{F}^*_{2,F}(E) \subseteq \mathfrak{F}^*_{2,F}(E \cup \{a\})$. Then Proposition 6 entails $\mathfrak{F}^*_F(E) = \mathfrak{F}^*_F(E \cup \{a\})$ and $|\text{Condef}_F(E \cup \{a\})| < |\text{Condef}_F(E)|$. Thus, $E \cup \{a\} \sqsupset^{\text{c-Condef}}_F E$. Therefore, strong reinstatement is satisfied.
- **"$\tau$ = Unatt":** Since $a \notin E \cup E^+_F$, $a \in \text{Unatt}(E)$ and adding $a$ does not remove any argument from $E^+_F$, $E^+_F \subseteq (E \cup \{a\})^+_F$. Thus, $|\text{Unatt}_F(E \cup \{a\})| < |\text{Unatt}_F(E)|$ and $E \cup \{a\} \sqsupset^{\text{c-Unatt}}_F E$. Therefore, strong reinstatement is satisfied. □

Lemma A.33. *$\sqsupseteq^{\text{c-}\tau}$ satisfies addition robustness for $\tau \in \{\text{Conflicts}, \text{Unatt}\}$.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ such that $E \sqsupseteq^{\text{c-}\tau}_F E'$ with respect to $\tau \in \{\text{Conflicts}, \text{Unatt}\}$. Consider AF $F' = (A, R \cup \{(a, b)\}$ such that $a \in E$, $b \notin E$, and $b \in E'$.

- **"$\tau$ = Conflicts":** Since $b \notin E$, $|\text{Conflicts}_F(E)| = |\text{Conflicts}_{F'}(E)|$, and $|\text{Conflicts}_F(E')| \leq |\text{Conflicts}_{F'}(E')|$, so if $E \sqsupseteq^{\text{c-Conflicts}}_F E'$ then $E \sqsupseteq^{\text{c-Conflicts}}_{F'} E'$.
- **"$\tau$ = Unatt":** For every attack $r$ in $R$, $r \in R \cup \{(a, b)\}$. Hence, for every argument $c \in A$, $c \neq a$, and $c^+_F = c^+_{F'}$. So, $|\text{Unatt}_F(E)| \leq |\text{Unatt}_{F'}(E)|$ and $|\text{Unatt}_F(E')| = |\text{Unatt}_{F'}(E')|$. Thus, if $E \sqsupseteq^{\text{c-Unatt}}_F E'$ then $E \sqsupseteq^{\text{c-Unatt}}_{F'} E'$. □

Lemma A.34. $\sqsupseteq^{c\text{-}\tau}$ *satisfies syntax independence for* $\tau \in \{\mathsf{Conflicts}, \mathsf{Undef}, \mathsf{Condef}, \mathsf{Unatt}\}$.

Proof. Clear by definition. □

□

**Proposition 15.** *For AF* $F = (A, R)$, $A \sqsupseteq^{\tau}_{F} E$ *for any* $E \subset A$ *with* $\tau \in \{\mathsf{nonatt}, \mathsf{strdef}\}$.

Proof. Let $F = (A, R)$ be any AF and $E \subset A$ a set of arguments.

**"$\sqsupseteq^{\mathsf{nonatt}}$":** For every argument $a \in E$, which not attacked by $A$ it has to hold that $a^{-}_{F} = \emptyset$, so $a$ has to be unattacked. Since $E \subset A$ and $a \in A$, therefore the number of unattacked arguments inside $E$ is smaller or equal to the number of unattacked arguments in $A$. Thus, $A \sqsupseteq^{\mathsf{nonatt}}_{F} E$ for every $E \subset A$.

**"$\sqsupseteq^{\mathsf{strdef}}$":** For set $E$ to contain any strongly defended arguments from $A$, $E$ needs to contain an unattacked argument. Like already discussed in the case above, $A$ contains every unattacked argument as well. Additionally, if an argument is strongly defended by $E$ from $A$, we can use the same reasoning to show the strong defence by $A$ from $E$. Therefore, the number of strongly defended arguments by $E$ is lower or equal to the number of strongly defended arguments in $A$. Hence, $A \sqsupseteq^{\mathsf{strdef}}_{F} E$ for every $E \subset A$. □

## A.4 Technical Proofs for Section 6

**Proposition 16.** r-ad *satisfies* ad*-generalisation, respect conflicts, composition, decomposition, weak reinstatement, addition robustness, and syntax independence.* r-ad *violates strong reinstatement.*

Proof. We consider each principle as follows.

Lemma A.35. r-ad *satisfies* ad*-generalisation.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. To show ad-generalisation is satisfied, we prove that if and only if $\mathsf{Conflicts}_F(E) = \mathsf{Undef}_F(E) = \emptyset$ then $E \in \mathsf{ad}(F)$.

**"ad-soundness":** First, $\emptyset$ is always an admissible set and therefore $\mathsf{Conflicts}_F(\emptyset) = \mathsf{Undef}_F(\emptyset) = \emptyset$, hence for every set $E$ that is a most plausible set according to r-ad, $\mathsf{Conflicts}_F(E) = \mathsf{Undef}_F(E) = \emptyset$, otherwise $\emptyset \sqsupset^{\text{r-ad}}_{F} E$. Thus, for every $E \in \max_{\text{r-ad}}(F)$, $E$ contains no conflict and every argument is defended, hence $E$ is admissible.

**"ad-completeness":** Suppose $E$ is admissible, then $E$ contains no conflicts implying $\mathsf{Conflicts}_F(E) = \emptyset$ and defends all its elements $\mathsf{Undef}_F(E) = \emptyset$. There cannot be a $E' \subseteq A$ such that $E' \sqsupset^{\mathsf{Conflicts}}_{F} E$ or $E' \sqsupset^{\mathsf{Undef}}_{F} E$, because there is no subset of $\emptyset$, implying $E \in \max_{\text{r-ad}}(F)$. □

Lemma A.36. r-ad *satisfies respect conflicts.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ such that $E \in \mathsf{cf}(F)$ and $E' \notin \mathsf{cf}(F)$. Then $E \sqsupset^{\mathsf{Conflicts}}_{F} E'$ and therefore $E \sqsupset^{\text{r-ad}}_{F} E'$. □

Lemma A.37. r-ad *satisfies composition and decomposition.*

Proof. Follows from Lemma A.22 together with Definition 26. □

Lemma A.38. r-ad *satisfies weak reinstatement.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. Suppose $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^{-} \cup E^{+})$. Then $\mathsf{Conflicts}_F(E \cup \{a\}) = \mathsf{Conflicts}_F(E)$. It remains to show: $\mathsf{Undef}_F(E \cup \{a\}) \subseteq \mathsf{Undef}_F(E)$. Suppose $x \in \mathsf{Undef}_F(E \cup \{a\})$. Then $x \in E \cup \{a\}$ and $x \notin \mathfrak{F}_F(E \cup \{a\})$. Because $a \in \mathfrak{F}_F(E \cup \{a\})$ it follows that $x \in E$. Furthermore, since $x \notin \mathfrak{F}_F(E \cup \{a\})$, $x \notin \mathfrak{F}_F(E)$. This implies that $x \in \mathsf{Undef}_F(E)$. Thus, $E \cup \{a\} \equiv^{\mathsf{Conflicts}}_{F} E$ and $E \cup \{a\} \sqsupseteq^{\mathsf{Undef}}_{F} E$, therefore $E \cup \{a\} \sqsupseteq^{\text{r-ad}}_{F} E$. □

Lemma A.39. r-ad *violates strong reinstatement.*

Proof. Follows directly from Proposition 5. □

Lemma A.40. r-ad *satisfies addition robustness.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ with $E \sqsupseteq_F^{\text{r-ad}} E'$. Let $F' = (A, R \cup \{(a, b)\}$ for $a \in E$, and $b \in E' \setminus E$.

**"$\sqsupseteq_F^{\text{Conflicts}}$":** The addition of $(a, b)$ does not add any new conflicts into $E$, $\text{Conflicts}_F(E) = \text{Conflicts}_{F'}(E)$, additionally no conflict from $E'$ is deleted, thus $\text{Conflicts}_F(E') \subseteq \text{Conflicts}_{F'}(E')$. Since $E \sqsupseteq_F^{\text{r-ad}} E'$, $\text{Conflicts}_F(E) \subseteq \text{Conflicts}_F(E')$, therefore $\text{Conflicts}_{F'}(E) \subseteq \text{Conflicts}_{F'}(E')$, implying $E \sqsupseteq_{F'}^{\text{Conflicts}} E'$.

**"$\sqsupseteq_F^{\text{Undef}}$":** Assume $E \sqsupseteq_F^{\text{r-ad}} E'$ and $\text{Conflicts}_F(E) = \text{Conflicts}_F(E')$. Like show above, $\text{Conflicts}_{F'}(E) \subseteq \text{Conflicts}_{F'}(E')$, assume $\text{Conflicts}_{F'}(E) = \text{Conflicts}_{F'}(E')$. Next, we show $\text{Undef}_{F'}(E) \subseteq \text{Undef}_{F'}(E')$. Since $E \sqsupseteq_F^{\text{r-ad}} E'$ and $\text{Conflicts}_F(E) = \text{Conflicts}_F(E')$, $\text{Undef}_F(E) \subseteq \text{Undef}_F(E')$. The attack $(a, b)$ does not disable any defence from $E$, since $b \notin E$. Hence, $b$ cannot be used to defend anything. Therefore, $\text{Undef}_{F'}(E) \subseteq \text{Undef}_F(E)$ and we do not lose any defence with respect to $E$.

Next, we show that $E'$ cannot defend more argument than before. Assume $b' \in \mathfrak{F}_{F'}(E')$ and $b' \notin \mathfrak{F}_F(E')$, meaning there is one attack $(c, b') \in R$ such that $E'$ does not attack $c$. Since only $(a, b)$ is added, this attack is responsible for the defends of $b'$ in $F'$, therefore $a \in E'$ has to hold. However, this is a contradiction to $\text{Conflicts}_{F'}(E) = \text{Conflicts}_{F'}(E')$, thus $b'$ cannot be defended. Therefore, $\text{Undef}_F(E') \subseteq \text{Undef}_{F'}(E')$ and since $\text{Undef}_F(E) \subseteq \text{Undef}_{F'}(E')$, $\text{Undef}_{F'}(E) \subseteq \text{Undef}_{F'}(E')$. Thus, $E \sqsupseteq_{F'}^{\text{r-ad}} E'$. □

Lemma A.41. r-ad *satisfies* syntax independence.

Proof. Clear by definition. □

□

**Proposition 17.** r-co *satisfies* co-*generalisation, respect conflicts, composition, decomposition, weak reinstatement, strong reinstatement, and syntax independence.* r-co *violates addition robustness.*

Proof. We consider each principle as follows.

Lemma A.42. r-co *satisfies* co-*generalisation.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$.

**"co-soundness":** Suppose $E \in \max_{\text{r-co}}(F)$. We first prove that $E$ is admissible. Suppose $E$ is not admissible, then there is an admissible $E'$ such that $E' \sqsupset_F^{\text{r-ad}} E$. But this contradicts $E \in \max_{\text{r-co}}(F)$, hence, $E$ is admissible. Next, we prove that $E$ contains all defended arguments. Suppose the contrary, then there is an argument $a \notin E$ and $a$ is defended by $E$. Thus, $a \in \text{Condef}_F(E)$. For some complete extension $E'$, $\text{Condef}_F(E') = \emptyset \subset \text{Condef}_F(E)$, implying $E' \equiv_F^{\text{r-ad}} E$ and $E' \sqsupset_F^{\text{r-co}} E$, contradicting $E \in \max_{\text{r-co}}(F)$. Hence, $E$ contains all arguments it defends. It follows that $E$ is complete.

**"co-completeness":** If $E$ is a complete extension of $F$. Suppose, towards contradiction, that $E \notin \max_{\text{r-co}}(F)$. Then there is an $E'$ such that $E' \sqsupset_F^{\text{r-co}} E$. Since $E$ is admissible, it follows that $E'$ is admissible. This implies $E' \sqsupset_F^{\text{Condef}} E$ and hence $\text{Condef}_F(E') \subset \text{Condef}_F(E)$. Consequently, there is an argument $a \in \text{Condef}_F(E)$, implying that there is an argument defended by $E$ but not element of $E$, this contradicts the assumption that $E$ is complete. Thus, $E \in \max_{\text{r-co}}(F)$. □

Lemma A.43. r-co *satisfies respect conflicts.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ such that $E \in \text{cf}(F)$ and $E' \notin \text{cf}(F)$. Then Conflicts implies, $E \sqsupset_F^{\text{Conflicts}} E'$, therefore $E \sqsupset_F^{\text{r-co}} E'$. □

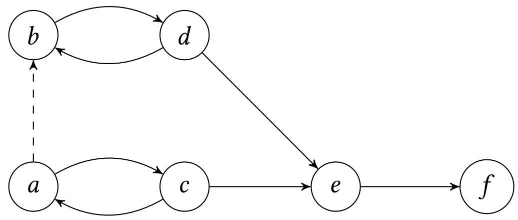


Fig. 32. AF $F_{16}$ from Example 57, the dashed attack from $a$ to $b$ is added to obtain $F'_{16}$.

Lemma A.44. r-co *satisfies composition and decomposition.*

Proof. Follows from Lemma A.22 together with Definition 27. □

Lemma A.45. r-co *satisfies strong reinstatement.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. Let $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^- \cup E^+)$. Lemma A.38 implies that $\{a\} \cup E \sqsupseteq_F^{\text{r-ad}} E$. If $\{a\} \cup E \sqsupset_F^{\text{r-ad}}, E$ we are done. Hence, we suppose $\{a\} \cup E \equiv_F^{\text{r-ad}} E$ and prove that $\text{Condef}_F(\{a\} \cup E) \subset \text{Condef}_F(E)$. Since $a \in \mathfrak{F}_F(E)$ and $a \notin \{E^- \cup E^+\}$, $a \in \mathfrak{F}^*(E)$ and $a \in \mathfrak{F}^*(E \cup \{a\})$. $\mathfrak{F}^*$ reaches a fixed point, implying $\mathfrak{F}^*(\{a\} \cup E) = \mathfrak{F}^*(E)$. Since $a \notin E$, $\mathfrak{F}^*(\{a\} \cup E) \setminus (E \cup \{a\}) \subset \mathfrak{F}^*(E) \setminus E$ and thus $\text{Condef}_F(\{a\} \cup E) \subset \text{Condef}_F(E)$. This implies $E \cup \{a\} \sqsupset_F^{\text{r-co}} E$. □

Lemma A.46. r-co *violates addition robustness.*

Example 57. *Let $F_{16}$ be the AF depicted in Figure 32. Consider the two sets $\{a\}$ and $\{b, c\}$. $\{a\}$ is a complete extension, while $\{b, c\}$ is not a complete extension, so:*

$$\{a\} \sqsupset_{F_{16}}^{\text{r-co}} \{b, c\}.$$

*If* addition robustness *is satisfied and we add an attack between $\{a\}$ and $\{b, c\}$, this maintains the relationship. Suppose we add $(a, b)$ to $F_{16}$ to create $F'_{16}$, as shown with the dashed line in Figure 32. Then $\{a\}$ is no longer a complete extension, since the arguments $d$ and $f$ are defended, implying* $\text{Condef}_{F'_{16}}(\{a\}) = \{d, f\}$, *while for $\{b, c\}$ we have* $\text{Condef}_{F'_{16}}(\{b, c\}) = \{f\}$. *Therefore:*

$$\{b, c\} \sqsupset_{F'_{16}}^{\text{r-co}} \{a\}.$$

*This violates addition robustness.*

Lemma A.47. r-co *satisfies* syntax independence.

Proof. Clear by definition. □

□

**Proposition 18.** r-gr *satisfies* gr*-generalisation, respect conflicts, composition, decomposition, weak reinstatement, strong reinstatement, and syntax independence.* r-gr *violates additiond robustness.*

Proof. We consider each principle as follows.

Lemma A.48. r-gr *satisfies* gr*-generalisation.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$.

**"gr-soundness":** Suppose $E \in \mathtt{max}_{\text{r-gr}}(F)$, then $E$ is complete. Suppose $E$ is not grounded. For the grounded extension $E'$ of $F$, we have $E' \subset E$, which implies $E' \sqsupset_F^{\text{r-gr}} E$. But this contradicts $E \in \mathtt{max}_{\text{r-gr}}(F)$. Hence, $E$ is the grounded extension.

"gr-**completeness**": Suppose $E$ is the grounded extension of $F$ and $E' \sqsupset_F^{\text{r-gr}} E$. Since $E$ is complete, $E'$ is also complete. Hence, $E' \subset E$, but this is impossible as it implies, $E$ is not the grounded extension. Thus, $E \in \max_{\text{r-gr}}(F)$. □

Lemma A.49. r-gr *satisfies respect conflicts, composition, decomposition, and strong reinstatement.* r-gr *violates addition robustness.*

Proof. **respect conflicts:** Follows directly from Lemma A.43.
**composition and decomposition:** Follows from Lemma A.22 together with Definition 28.
**strong reinstatement:** Follows directly from Lemma A.45.
**addition robustness:** We can use Example 57, to show that r-gr violates *addition robustness.* □

Lemma A.50. r-gr *satisfies syntax independence.*

Proof. Clear by definition. □

□

**Proposition 19.** r-pr *and* r-co-pr *satisfy* pr-*generalisation, respect conflicts, composition, decomposition,* strong reinstatement*, syntax independence, and violates addition robustness.*

Proof. We consider each principle as follows.

Lemma A.51. r-pr *and* r-co-pr *are satisfying* pr-*generalisation.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. We start with r-pr.

"pr-**soundness**": Assume $E \in \max_{\text{r-pr}}(F)$. Then $E$ is an admissible set. Suppose $E$ is not preferred. Then there is an admissible set $E'$ such that $E \subset E'$. But this is a contradiction to $E \in \max_{\text{r-pr}}(F)$. Thus, $E$ is a preferred extension.

"pr-**completeness**": Suppose $E$ is a preferred extension of $F$ and assume $E' \sqsupset_F^{\text{r-pr}} E$. Since $E$ is an admissible set, $E'$ is also an admissible set. Hence, we have $E \subset E'$. But this is impossible as it implies that $E$ is not a preferred extension. Thus, $E \in \max_{\text{r-pr}}(F)$.

Next, we look at r-co-pr.

"pr-**soundness**": Suppose $E \in \max_{\text{r-co-pr}}(F)$. Then $E$ is complete. Assume $E$ is not preferred. Then there is an complete $E'$ such that $E \subset E'$. But this is a contradiction to $E \in \max_{\text{r-co-pr}}(F)$. Thus, $E$ is a preferred extension.

"pr-**completeness**": Suppose $E$ is a preferred extension of $F$ and assume $E' \sqsupset_F^{\text{r-co-pr}} E$. Since $E$ is complete, $E'$ is also complete. Hence, we have $E \subset E'$. But this is impossible as it implies that $E$ is not a preferred extension. Thus, $E \in \max_{\text{r-co-pr}}(F)$. □

Lemma A.52. r-pr *and* r-co-pr *are satisfying respect conflicts.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ such that $E \in \text{cf}(F)$ and $E' \notin \text{cf}(F)$. Then $E \sqsupset_F^{\text{Conflicts}} E'$, therefore $E \sqsupset_F^{\text{r-pr}} E'$ and $E \sqsupset_F^{\text{r-co-pr}} E'$ □

Lemma A.53. r-pr *and* r-co-pr *are satisfying composition and decomposition.*

Proof. Follows from Lemma A.22 together with Definition 29 and Definition 30, respectively. □

Lemma A.54. r-pr *and* r-co-pr *are satisfying strong reinstatement.*

Proof. We only show it for r-pr, but the proof for r-co-pr is similar.
Let $F = (A, R)$ be an AF and $E \subseteq A$. Let $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^- \cup E^+)$. Lemma A.38 implies, that $E \cup \{a\} \sqsupseteq_F^{\text{r-pr}} E$. It also hold that $E \subset E \cup \{a\}$. Hence, $E \cup \{a\} \sqsupset_F^{\text{r-pr}} E$. So, r-pr satisfies strong reinstatement. □

Lemma A.55. r-pr *and* r-co-pr *are violating addition robustness.*

Proof. To show that addition robustness is violated by r-pr and r-co-pr, we use Example 50 again. □

Lemma A.56. r-pr *and* r-co-pr *are satisfying syntax independence.*

Proof. Clear by definition. □

□

**Proposition 20.** *Let $F = (A, R)$ be an AF with $\mathsf{st}(F) \neq \emptyset$, then $\max_{\text{r-sst}}(F) = \mathsf{st}(F)$.*

Proof. Let $F = (A, R)$ be an AF such that $\mathsf{st}(F) \neq \emptyset$.

- **"⊆":** Let $E \in \max_{\text{r-sst}}(F)$ and $E' \in \mathsf{st}(F)$, meaning that $\mathsf{Conflicts}_F(E) = \mathsf{Undef}_F(E) = \mathsf{Condef}_F(E) = \emptyset$ and $E' \in \mathsf{co}(F)$, implying $\mathsf{Conflicts}_F(E') = \mathsf{Undef}_F(E') = \mathsf{Condef}_F(E') = \emptyset$. Additionally, $E'$ attacks every argument not inside, hence $\mathsf{Unatt}_F(E') = \emptyset$, so for $E \in \max_{\text{r-sst}}(F)$ it has to hold that $\mathsf{Unatt}_F(E) = \emptyset$, meaning that $E$ is conflict-free and attacks everything not inside, implying that $E$ is a stable extension.
- **"⊇":** Let $E \in \mathsf{st}(F)$, this means that $E$ is conflict-free and attacks every argument not in $E$, i. e., $\mathsf{Conflicts}_F(E) = \mathsf{Unatt}_F(E) = \emptyset$. Additionally, $E$ is complete, therefore $\mathsf{Undef}_F(E) = \mathsf{Condef}_F(E) = \emptyset$. There cannot be any set be ranked strictly more plausible to be accepted than $E$ with respect to r-sst, thus $E \in \max_{\text{r-sst}}(F)$. □

**Proposition 21.** r-sst *satisfies* st*-completeness.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. If $\mathsf{st}(F) = \emptyset$, then $\max_{\text{r-sst}}(F) \supseteq \emptyset$.
Suppose $E \in \mathsf{st}(F)$, then $\mathsf{Conflicts}_F(E) = \emptyset$ and $\mathsf{Unatt}_F(E) = \emptyset$, implying that $E \in \max_{\text{r-sst}}(F)$, thus st-completeness is satisfied. □

**Proposition 22.** r-sst *satisfies* sst*-generalisation, respect conflicts, composition, decomposition, strong reinstatement, and syntax independence.* r-sst *violates addition robustness.*

Proof. We consider each principle as follows.

Lemma A.57. r-sst *satisfies* sst*-generalisation.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$.

- **"sst-soundness":** Suppose $E \in \max_{\text{r-sst}}(F)$. Then $E$ is complete. Assume $E$ is not semi-stable. Then there is a complete set $E'$ such that $E \cup E^+ \subset E' \cup E'^+$. But, this implies $E' \sqsupset_F^{\mathsf{Unatt}} E$, contradicting $E \in \max_{\text{r-sst}}(F)$. Hence, $E$ is a semi-stable extension.
- **"sst-completeness":** Suppose $E$ is a semi-stable extension of $F$ and assume $E' \sqsupset_F^{\text{r-sst}} E$. Since $E$ is complete, $E'$ is also complete. Hence, $E' \sqsupset_F^{\mathsf{Unatt}} E$. But this is impossible as it implies that $E \cup E^+ \subset E' \cup E'^+$, meaning that $E$ is not a semi-stable extension. Thus, $E \in \max_{\text{r-sst}}(F)$. □

Lemma A.58. r-sst *satisfies respect conflicts, composition, decomposition, and strong reinstatement.* r-sst *violates* addition robustness.

Proof. **respect conflicts:** Follows directly from Lemma A.43.
**composition and decomposition:** Follows from Lemma A.22 together with Definition 31.

**strong reinstatement:** Follows directly from Lemma A.45.
**addition robustness:** We can use Example 57, to show that r-sst violates addition robustness. □

Lemma A.59. r-sst *satisfies syntax independence.*

Proof. Clear by definition. □

□

**Proposition 23.** *Let $F = (A, R)$ be an AF and $E, E' \subseteq A$. If $E \sqsupseteq_F^{\text{r-}\sigma} E'$, then $E \sqsupseteq_F^{\text{r-c-}\sigma} E'$ for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$, assume $E \sqsupseteq_F^{\text{r-}\sigma} E'$ for $\sigma \in \{\text{ad}, \text{co}, \text{sst}\}$, then there is one $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$ such that $\tau_F(E) \subseteq \tau_F(E')$, this subset behaviour entails $|\tau_F(E)| \leq |\tau_F(E')|$, thus $E \sqsupseteq_F^{\text{r-c-}\sigma} E'$.

For $\sigma \in \{\text{gr}, \text{pr}, \text{co-pr}\}$, we still use $\subseteq$ and $\supseteq$, respectively, so there is no difference between these variants. □

**Proposition 24.** r-c-$\sigma$ *satisfies and violates the principles as depicted in Table 5.*

Proof. We consider each principle as follows.

Lemma A.60. r-c-$\sigma$ *satisfies $\sigma$-generalisation for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.*

Proof. For any $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$, the best value a set can have is $\emptyset$. For the cardinality-based versions of these relations, the best value to be returned is 0, and only the empty set can have cardinality of 0. Therefore, we know that if and only if $\tau$ returns $\emptyset$, the cardinality-based versions of r-c-$\sigma$ returns 0. Thus, we can use the same proof ideas of r-$\sigma$ to show that r-c-$\sigma$ satisfies $\sigma$-generalisation. □

Lemma A.61. r-c-$\sigma$ *satisfies respect conflicts for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ such that $E \in \text{cf}(F)$ and $E' \notin \text{cf}(F)$. Then $E \sqsupset_F^{\text{c-Conflicts}} E'$ and $E \sqsupset_F^{\text{r-c-}\sigma} E'$ for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$. □

Lemma A.62. r-c-$\sigma$ *satisfies composition for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.*

Proof. Let $F_1 = (A_1, R_1)$, $F_2 = (A_2, R_2)$ and $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ be AFs with $A_1 \cap A_2 = \emptyset$. We know that $|\tau_{F_1}(E \cap A_1)| + |\tau_{F_2}(E \cap A_2)| = |\tau_F(E)|$ for any base relation $\tau \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$. Assume $E \cap A_1 \sqsupseteq_{F_1}^{\text{r-c-}\sigma} E' \cap A_1$ and $E \cap A_2 \sqsupseteq_{F_2}^{\text{r-c-}\sigma} E' \cap A_2$. Thus, there is at least one $\tau' \in \{\text{Conflicts}, \text{Undef}, \text{Condef}, \text{Unatt}\}$ such that $|\tau'_{F_1}(E \cap A_1)| \leq |\tau'_{F_1}(E' \cap A_1)|$ and $|\tau'_{F_2}(E \cap A_2)| \leq |\tau'_{F_2}(E' \cap A_2)|$. Therefore, $|\tau'_{F_1}(E \cap A_1)| + |\tau'_{F_2}(E \cap A_2)| \leq |\tau'_{F_1}(E' \cap A_1)| + |\tau'_{F_2}(E' \cap A_2)|$ and this entails $|\tau'_F(E)| \leq |\tau'_F(E')|$. Hence, composition is satisfied. □

Lemma A.63. r-c-$\sigma$ *violates decomposition for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.*

Example 58. *Consider $F_{17} = (\{a, b, c, d, e\}, \{(a, b), (a, c), (d, e)\})$ as depicted in Figure 33, with $\{d, e\}$ and $\{a, b, c\}$. This AF can be split into two disjoint subAFs: $F_{17,1} = (\{a, b, c\}, \{(a, b), (a, c)\})$ and $F_{17,2} = (\{d, e\}, \{(d, e)\})$. Since $\{a, b, c\}$ has two conflicts and $\{d, e\}$ has only one, it is clear that:*

$$\{d, e\} \sqsupset_{F_{17}}^{\text{c-Conflicts}} \{a, b, c\}.$$

*Given that every* r-c-$\sigma$ *is based on $\sqsupseteq^{\text{c-Conflicts}}$, the same holds for $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{sst}\}$.*

*Examining the subAFs $F_{17,1}$ and $F_{17,2}$, we see that $\{d, e\} \cap \{a, b, c\}$ is conflict-free in $F_{17,1}$, while $\{a, b\} \cap \{a, b, c\}$ has two conflicts, so:*

$$\{d, e\} \cap \{a, b, c\} \sqsupset_{F_{17,1}}^{\text{c-Conflicts}} \{d, e\} \cap \{a, b, c\}.$$

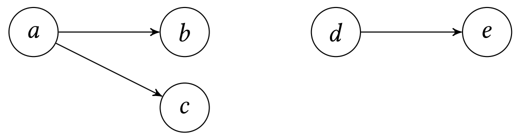


Fig. 33. AF $F_{17}$ from Example 58.

*However, $E' \cap \{d, e\}$ is conflict-free in $F_{17,2}$ and $E \cap \{d, e\}$ has one conflict in $F_{17,2}$, so:*

$$\{a, b, c\} \cap \{d, e\} \sqsupset^{\text{c-Conflicts}}_{F_{17,2}} \{d, e\} \cap \{d, e\}.$$

*This shows that* c-Conflicts *violates decomposition, thus* r-c-$\sigma$ *for* $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{sst}\}$ *also violates decomposition.*

Lemma A.64. r-c-ad *satisfies weak reinstatement and violates strong reinstatement.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. Suppose $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin E^- \cup E^+$. In Lemma A.23, we have shown that $\text{Undef}_F(E \cup \{a\}) \subseteq \text{Undef}_F(E)$ and therefore also $|\text{Undef}_F(E \cup \{a\})| \leq |\text{Undef}_F(E)|$. This proves that r-c-ad satisfies weak reinstatement.

Since ad-generalisation is satisfied, we know that strong reinstatement is violated. □

Lemma A.65. r-c-$\sigma$ *satisfies strong reinstatement for* $\sigma \in \{\text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. Suppose $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin E^- \cup E^+$. In Lemma A.23, we have shown that $\text{Condef}_F(E \cup \{a\}) \subset \text{Condef}_F(E)$. This also shows that $|\text{Condef}_F(E \cup \{a\})| < |\text{Condef}_F(E)|$. Hence, r-c-$\sigma$ satisfies strong reinstatement and because $E \cup \{a\} \sqsupset^{\text{r-c-}\sigma}_F E$, this also shows the satisfaction of strong reinstatement for the remaining semantics $\sigma \in \{\text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$. □

Lemma A.66. r-c-ad *satisfies addition robustness.*

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$ with $E \sqsupseteq^{\text{r-c-ad}}_F E'$. Let $F' = (A, R \cup \{(a, b)\})$ for $a \in E$ and $b \notin E$, $b \in E'$. The addition of $(a, b)$ does not add any new conflict into $E$ and does not remove any conflict from $E'$, therefore $|\text{Conflicts}_F(E)| = |\text{Conflicts}_{F'}(E)|$ and $|\text{Conflicts}_F(E')| \leq |\text{Conflicts}_{F'}(E')|$. Hence, addition robustness is satisfied by $\sqsupseteq^{\text{c-Conflicts}}$.

It remains to show, that addition robustness holds for $\sqsupseteq^{\text{c-Undef}}$ as well. We already know that no defence in $E$ is removed, therefore $|\text{Undef}_{F'}(E)| \leq |\text{Undef}_F(E)|$ and $E'$ cannot defend more arguments in $F'$ than in $F$, otherwise additional conflicts have to be added. Thus, $E \sqsupseteq^{\text{r-c-ad}}_{F'} E'$. □

Lemma A.67. r-c-$\sigma$ *violates addition robustness for* $\sigma \in \{\text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.

Example 59. *For* $\sigma \in \{\text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$, *the counterexamples used before (Examples 50 and 57) can be reused to show that* r-c-$\sigma$ *also violates addition robustness. For instance, in Example 57, it holds that* $\text{Condef}_{F_{16}}(\{a\}) = \emptyset$ *and* $\text{Condef}_{F_{16}}(\{b, c\}) = \{d\}$. *After adding an attack from* $\{a\}$ *to* $\{b, c\}$, *the consistently defended arguments of* $\{a\}$ *changed to* $\text{Condef}_{F'_{16}}(\{a\}) = \{d, f\}$, *while the consistently defended arguments of* $\{b, c\}$ *remained the same, i.e.,* $\text{Condef}_{F'_{16}}(\{b, d\}) = \{d\}$. *Thus, the relation between* $\{a\}$ *and* $\{b, c\}$ *in* $F'_{16}$ *changes to:*

$$\{b, d\} \sqsupset^{\text{r-c-}\sigma}_{F'_{16}} \{a\}.$$

*Therefore,* r-c-$\sigma$ *violates addition robustness for* $\sigma \in \{\text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.

Lemma A.68. r-c-$\sigma$ *satisfies syntax independence for* $\sigma \in \{\text{ad}, \text{co}, \text{gr}, \text{pr}, \text{co-pr}, \text{sst}\}$.

Proof. Clear by definition. □

□

**Proposition 25.** *Let $F = (A, R)$ be an AF. $\sqsupseteq_F^{\text{cope}(\tau_1,\ldots,\tau_n)}$ is reflexive and transitive for any sequence of base relations $(\tau_1, \ldots, \tau_n)$.*

Proof. Let $F = (A, R)$ be an AF and $(\tau_1, \ldots, \tau_n)$ a sequence of base relations.

**"reflexive":** For any $E \subseteq A$ it has to hold that $E \equiv_F^{\text{cope}(\tau_1,\ldots,\tau_n)} E$. Let $w_i$ be the number of sets ranked worse than $E$ with respect to $\sqsupseteq_F^{\tau_i}$ and $l_i$ be the number of sets ranked better than $E$ with respect to $\sqsupseteq_F^{\tau_i}$, then for $E \equiv_F^{\text{cope}(\tau_1,\ldots,\tau_n)} E$ to hold, $w_i - l_i = w_i - l_i$ has to hold for every $1 \leq i \leq n$, which is clear by definition.

**"transitive":** Let $E, E', E'' \subseteq A$ and $E \sqsupset_F^{\text{cope}(\tau_1,\ldots,\tau_n)} E'$ and $E' \sqsupset_F^{\text{cope}(\tau_1,\ldots,\tau_n)} E''$, then $E$ has a better win/loss record than $E'$ and $E'$ has a better win/loss record than $E''$. Let $w$, $w'$, $w''$ be the corresponding number of wins minus the number of loses of $E$, $E'$ and $E''$, respectively. Thus, $w > w'$ and $w' > w''$, by definition $w > w''$, which shows that $E \sqsupset_F^{\text{cope}(\tau_1,\ldots,\tau_n)} E''$. □

**Proposition 26.** *Let $F = (A, R)$ be an AF. If base relation $\sqsupseteq^\tau$ is total and transitive, then* $\text{cope}(\sqsupseteq_F^\tau) = \sqsupseteq_F^\tau$.

Proof. Let $F = (A, R)$ be an AF and $E, E' \subseteq A$. Let $\sqsupseteq^\tau$ be a base relation that is total and transitive. First, we show that if $E \sqsupseteq_F^\tau E'$ then $E \sqsupseteq_F^{\text{cope}(\tau)} E'$. Assume $E \sqsupseteq_F^\tau E'$, then for every $E''$ such that $E' \sqsupseteq_F^\tau E''$, because of transitivity, we have $E \sqsupseteq_F^\tau E''$. So, $E$ wins at least as many times as $E'$. If $E'' \sqsupseteq_F^\tau E$, then $E'' \sqsupseteq_F^\tau E'$, so $E'$ loses at least as often as $E$. Thus, $|\{\mathcal{E} \subseteq A | E \sqsupseteq_F^\tau \mathcal{E}\}| - |\{\mathcal{E}' \subseteq A | \mathcal{E}' \sqsupseteq_F^\tau E\}| \geq |\{\mathcal{E} \subseteq A | E' \sqsupseteq_F^\tau \mathcal{E}\}| - |\{\mathcal{E}' \subseteq A | \mathcal{E}' \sqsupseteq_F^\tau E'\}|$, implying $E \sqsupseteq_F^{\text{cope}(\tau)} E'$.

Next, we show the other direction. Let $E \sqsupseteq_F^{\text{cope}(\tau)} E'$, and assume for contrary $E' \sqsupset_F^\tau E$, similarly to what we showed above, this implies $E' \sqsupseteq_F^{\text{cope}(\tau)} E$ and because of transitivity and totality of $\tau$, the relationship has to be strict, $E' \sqsupset_F^{\text{cope}(\tau)} E$. Thus, $E \sqsupseteq_F^{\text{cope}(\tau)} E'$ can only imply $E \sqsupseteq_F^\tau E'$ or $E \asymp_F^\tau E'$. However, the second option is not possible since $\sqsupseteq_F^\tau$ is total. So, $E \sqsupseteq_F^{\text{cope}(\tau)} E'$ implies $E \sqsupseteq_F^\tau E'$. □

**Proposition 27.** $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$, $\text{cope}(\sqsupseteq^{\text{nonatt}})$ *and* $\text{cope}(\sqsupseteq^{\text{strdef}})$ *satisfy and violated the principles as depicted in Table 6.*

Proof. We consider each principle as follows.

Lemma A.69. $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ *satisfies* ad*-generalisation.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$.

"ad-**completeness**": Suppose $E$ is admissible, then $\text{Conflicts}_F(E) = \text{Undef}_F(E) = \emptyset$ and therefore $E \sqsupseteq_F^{\text{Conflicts}} E'$ and $E \sqsupseteq_F^{\text{Undef}} E'$ for any $E' \subseteq A$. So, $E$ wins against every set of arguments and $E$ only loses against sets $\mathcal{E}$ with $\text{Conflicts}_F(\mathcal{E}) = \emptyset$ and $\text{Undef}_F(\mathcal{E}) = \emptyset$, respectively. However, all these sets are also losing against $E$, there is no set with less loses than $E$. $E$ wins the maximal amount of times and no set loses less than $E$, therefore there cannot be any set with a better win/ loss record than $E$, implying $E \in \max_{\text{cope}(\text{Conflicts},\text{Undef})}(F)$.

"ad-**soundness**": Suppose $E \in \max_{\text{cope}(\text{Conflicts},\text{Undef})}(F)$, so $E \sqsupseteq_F^{\text{cope}(\text{Conflicts},\text{Undef})} \emptyset$. We know that $\text{Conflicts}(\emptyset) = \text{Undef}(\emptyset) = \emptyset$, meaning that $\emptyset$ wins the maximal amount of times and there is no set which loses less than $\emptyset$, i. e., $\emptyset \in \max_{\text{cope}(\text{Conflicts},\text{Undef})}(F)$. For $E \in \max_{\text{cope}(\text{Conflicts},\text{Undef})}(F)$ to hold $E$ has to have the same win/ loss record as $\emptyset$, which is only possible if $\text{Conflicts}(E) = \text{Undef}(E) = \emptyset$, implying that $E$ is admissible. □

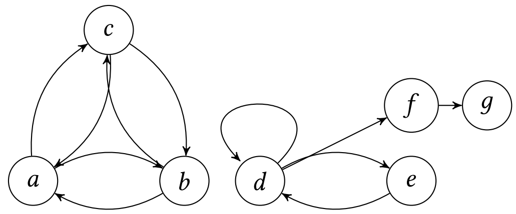


Fig. 34. AF $F_{18}$ from Example 60.

Lemma A.70. $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ *violates respect conflicts and composition.*

Example 60. *Let* $F_{18} = (\{a,b,c,d,e,f,g\}, \{(a,b),(b,a),(c,a),(a,c),(b,c),(c,b),(d,d),(d,e),(e,d),(d,f),(f,g)\}$ *be an AF, as depicted in Figure 34.* $F_{18}$ *can be split into two disjoint AFs* $F_{18,1} = (\{a,b,c\}, \{(a,b),(b,a),(a,c),(b,c),(c,b)\})$ *and* $F_{18,2} = (\{d,e,f,g\}, \{(d,d),(d,e),(e,d),(d,f),(f,g)\}$. *Consider the sets* $\{a,d,e\}$ *and* $\{b,e,g\}$*, then we get:*

$$\{a\} \equiv^{\text{cope(Conflicts,Undef)}}_{F_{18,1}} \{b\} \quad \text{and} \quad \{e,g\} \sqsupset^{\text{cope(Conflicts,Undef)}}_{F_{18,2}} \{d,e\}.$$

*Thus,* $\{b,e,g\}$ *should be at least as plausible to be accepted as* $\{a,d,e\}$ *in* $F_{18}$ *with respect to* $\sqsupseteq^{\text{cope(Conflicts,Undef)}}_{F_{18}}$. *However, we find:*

$$\{a,d,e\} \sqsupset^{\text{cope(Conflicts,Undef)}}_{F_{18}} \{b,e,g\}.$$

*The set* $\{a,d,e\}$ *is not conflict-free due to the self-attacking argument* $d$*, whereas* $\{b,e,g\}$ *is conflict-free. Therefore, respect conflicts is also violated.*

Lemma A.71. $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ *violates decomposition.*

Example 61. *Consider AF* $F_{17}$ *from Example 58, which can be partitioned into* $F_{17,1} = (\{a,b,c\}, \{(a,b),(a,c)\})$ *and* $F_{17,2} = (\{d,e\}, \{(d,e)\})$. *Consider the sets* $\{a,e\}$ *and* $\{c,d\}$. *We have:*

$$\{c,d\} \equiv^{\text{cope(Conflicts,Undef)}}_{F_{17}} \{a,e\} \quad \text{and} \quad \{d\} \sqsupset^{\text{cope(Conflicts,Undef)}}_{F_{17,2}} \{e\}.$$

*However, we also have:*

$$\{a\} \sqsupset^{\text{cope(Conflicts,Undef)}}_{F_{17,1}} \{c\}.$$

*Thus, violating decomposition.*

Lemma A.72. $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ *satisfies weak reinstatement and violates strong reinstatement.*

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. Consider $a \in \mathcal{F}_F$, $a \notin E$, and $a \notin (E^- \cup E^+)$. First, we examine $\sqsupseteq^{\text{Conflicts}}_F$ and $\sqsupseteq^{\text{Undef}}_F$ one by one.

- **"$\sqsupseteq^{\text{Conflicts}}$":** Since $a \notin (E^- \cup E^+)$, $\text{Conflicts}_F(E) = \text{Conflicts}_F(E \cup \{a\})$. Hence, if $E \sqsupseteq^{\text{Conflicts}}_F E'$ for any $E' \subseteq A$ then $E \cup \{a\} \sqsupseteq^{\text{Conflicts}}_F E'$ holds as well. Thus, the win/ loss record of $E$ and $E \cup \{a\}$ are the same.
- **"$\sqsupseteq^{\text{Undef}}$":** Since $a \in \mathcal{F}_F(E)$, $\text{Undef}(E) = \text{Undef}(E \cup \{a\})$. Thus, again the win/ loss records of these two sets are equal.

Next, let $w_{\text{Conflicts}}$ and $w_{\text{Undef}}$ be the win/ loss records of $E$ and $w'_{\text{Conflicts}}$ and $w'_{\text{Undef}}$ the win/ loss records of $E \cup \{a\}$. Since $w_{\text{Conflicts}} = w'_{\text{Conflicts}}$ and $w_{\text{Undef}} = w'_{\text{Undef}}$, it holds that $w_{\text{Conflicts}} + w_{\text{Undef}} = w'_{\text{Conflicts}} + w'_{\text{Undef}}$. Thus, $E \cup \{a\} \equiv^{\text{cope(Conflicts,Undef)}}_F E$ and weak reinstatement is satisfied and strong reinstatement is violated. □

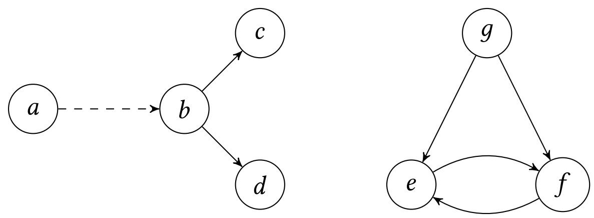


Fig. 35. AF $F_{19}$ from Example 62, where attack $(a, b)$ is added later to obtain $F'_{19}$.

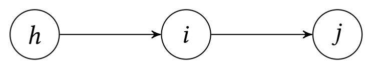


Fig. 36. Recalled AF $F_3$ from Example 10.

Lemma A.73. $\mathrm{cope}(\sqsupseteq^{\mathsf{Conflicts}}, \sqsupseteq^{\mathsf{Undef}})$ *violates additional robustness.*

Example 62. *Let* $F_{19} = (A_{19}, R_{19}) = (\{a, b, c, d, e, f, g\}, \{(b, c), (b, d), (g, f), (g, e), (f, e), (e, f)\})$ *be an AF, as depicted in Figure 35. Consider sets* $\{a, b, c, d\}$ *and* $\{a, e, f\}$*. We have:*

$$\{a, e, f\} \sqsupset_{F_{19}}^{\mathrm{cope}(\mathsf{Conflicts},\mathsf{Undef})} \{a, b, c, d\}.$$

*Adding an attack between a and b results in AF* $F'_{19} = (A_{19}, R_{19} \cup \{(a, b)\})$*. If additional robustness were satisfied, it would hold that:*

$$\{a, e, f\} \sqsupset_{F'_{19}}^{\mathrm{cope}(\mathsf{Conflicts},\mathsf{Undef})} \{a, b, c, d\}.$$

*But this is not the case since:*

$$\{a, b, c, d\} \sqsupset_{F'_{19}}^{\mathrm{cope}(\mathsf{Conflicts},\mathsf{Undef})} \{a, e, f\}.$$

*Hence, additional robustness is violated.*

Lemma A.74. $\mathrm{cope}(\sqsupseteq^{\tau})$ *violates* $\sigma$*-generalisation for* $\sigma \in \{\mathsf{cf}, \mathsf{ad}, \mathsf{co}, \mathsf{gr}, \mathsf{pr}, \mathsf{st}, \mathsf{sst}\}$ *and respect conflicts with* $\tau \in \{\mathsf{nonatt}, \mathsf{strdef}\}$.

Example 63. *Consider* $F_3 = (\{h, i, j\}, \{(h, i), (i, j)\})$ *from Example 10 (recalled in Figure 36). For both* $\sqsupseteq^{\mathrm{cope}(\mathsf{nonatt})}$ *and* $\sqsupseteq^{\mathrm{cope}(\mathsf{strdef})}$*, the set* $\{h, i, j\}$ *is among the most plausible sets, i. e.,* $\{h, i, j\} \in \mathtt{max}_{\mathrm{cope}(\tau)}(F_3)$ *for* $\tau \in \{\mathsf{nonatt}, \mathsf{strdef}\}$*. Hence,* $\sigma$*-generalisation and respect conflicts are violated for* $\sigma \in \{\mathsf{cf}, \mathsf{ad}, \mathsf{co}, \mathsf{gr}, \mathsf{pr}, \mathsf{st}, \mathsf{sst}\}$*.*

Lemma A.75. $\mathrm{cope}(\sqsupseteq^{\tau})$ *violates composition for* $\tau \in \{\mathsf{nonatt}, \mathsf{strdef}\}$.

Example 64. *Consider* $F_{20} = (\{a, b, c, d, e\}, \{(a, b), (b, a), (c, d), (c, e)\})$ *as depicted in Figure 37.* $F_{20}$ *can be partitioned into two disjoint AFs:* $F_{20,1} = (\{a, b\}, \{(a, b), (b, a)\})$ *and* $F_{20,2} = (\{c, d, e\}, \{(c, d), (c, e)\}$*. Consider the sets* $\{a, c\}$ *and* $\{b, c, d, e\}$*. In* $F_{20,1}$*,* $\{a\}$ *wins the comparison against* $\emptyset$ *and* $\{b\}$ *with respect to* $\sqsupseteq_{F_{20,1}}^{\mathsf{nonatt}}$ *and* $\sqsupseteq_{F_{20,1}}^{\mathsf{strdef}}$*, whereas* $\{b\}$ *wins the comparison against* $\emptyset$ *and* $\{a\}$*, thus* $\{a\}$ *and* $\{b\}$ *have the same win/lose record in* $F_{20,1}$ *with respect to* $\sqsupseteq_{F_{20,1}}^{\mathrm{cope}(\mathsf{nonatt})}$ *and* $\sqsupseteq_{F_{20,1}}^{\mathrm{cope}(\mathsf{strdef})}$*. In* $F_{20,2}$ *similar holds, for* $\tau \in \{\mathsf{nonatt}, \mathsf{strdef}\}$*, we get:*

$$\{a\} \equiv_{F_{20,1}}^{\mathrm{cope}(\tau)} \{b\} \quad \textit{and} \quad \{c\} \equiv_{F_{20,2}}^{\mathrm{cope}(\tau)} \{c, d, e\}.$$

*In* $F_{20}$*,* $\{a, c\}$ *wins 27 comparisons and loses 15 comparisons with respect to* $\sqsupseteq_{F_{20}}^{\mathsf{nonatt}}$ *and* $\sqsupseteq_{F_{20}}^{\mathsf{strdef}}$*,* $\{b, c, d, e\}$ *wins 27 comparisons and loses 11 comparisons with respect to* $\sqsupseteq_{F_{20}}^{\mathsf{nonatt}}$ *and* $\sqsupseteq_{F_{20}}^{\mathsf{strdef}}$*. Thus,* $\{b, c, d, e\}$ *has a better win/lose ratio*

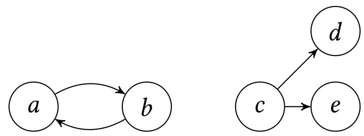


Fig. 37. AF $F_{20}$ from Example 64.

*than* $\{a, c\}$ *and therefore for* $\tau \in \{\text{nonatt}, \text{strdef}\}$*:*

$$\{b, c, d, e\} \sqsupset_{F_{20}}^{\text{cope}(\tau)} \{a, c\}.$$

*This shows, that composition is violated.*

Lemma A.76. $\text{cope}(\sqsupseteq^{\tau})$ *violates decomposition for* $\tau \in \{\text{nonatt}, \text{strdef}\}$.

Example 65. *Consider* $F_{17} = (\{a, b, c, d, e\}, \{(a, b), (a, c), (d, e)\})$ *be an AF from Example 17. This AF can be partitioned into* $F_{17,1} = (\{a, b, c\}, \{(a, b), (a, c)\})$ *and* $F_{17,2} = (\{d, e\}, \{(d, e)\})$*. Consider the sets* $\{a, b, e\}$ *and* $\{c, d\}$*. For* $\tau \in \{\text{nonatt}, \text{strdef}\}$*, we have:*

$$\{a, b, e\} \sqsupset_{F_{17}}^{\text{cope}(\sqsupseteq^{\tau})} \{c, d\}.$$

*We also have:*

$$\{a, b\} \sqsupset_{F_{17,1}}^{\text{cope}(\sqsupseteq^{\tau})} \{c\} \quad \textit{and} \quad \{d\} \sqsupset_{F_{17,2}}^{\text{cope}(\sqsupseteq^{\tau})} \{e\}.$$

Lemma A.77. $\text{cope}(\sqsupseteq^{\tau})$ *satisfies strong reinstatement for* $\tau \in \{\text{nonatt}, \text{strdef}\}$.

Proof. Let $F = (A, R)$ be an AF and $E \subseteq A$. Consider $a \in \mathfrak{F}_F(E)$, $a \notin E$, and $a \notin (E^- \cup E^+)$.

**"$\tau$ = nonatt":** We show that if $E \sqsupseteq_F^{\text{nonatt}} E'$ then $E \cup \{a\} \sqsupseteq_F^{\text{nonatt}} E'$ for $E' \subseteq A$. Every argument that is attacked by $E$ is also attacked by $E \cup \{a\}$, i.e., $E^+ \subseteq (E \cup \{a\})^+$. If $a$ is attacked by $E'$, since $a \in \mathfrak{F}_F(E)$, the attacker of $a$ is attacked by $E$, hence the addition of $a$ does not add any new attacked arguments, meaning $E \cup \{a\} \sqsupseteq_F^{\text{nonatt}} E'$ if $E \sqsupseteq_F^{\text{nonatt}} E'$, implying that $E \cup \{a\}$ wins at least as often as $E$.
Next, we show that $E$ loses at least as often as $E \cup \{a\}$. Assume $E' \sqsupseteq_F^{\text{nonatt}} E \cup \{a\}$, then like discussed before every argument attacked by $E'$ in $E \cup \{a\}$ is also attacked by $E'$ in $E$ except $a$, however since $a$ is defended by $E$ this argument does not create any problem, so $E' \sqsupseteq_F^{\text{nonatt}} E$. Thus, the win/lose ratio of $E \cup \{a\}$ is at least as good as $E$ and also $E \cup \{a\}$ wins against $E$, so $E \cup \{a\} \sqsupset_F^{\text{cope}(\text{nonatt})} E$.

**"$\tau$ = strdef":** Similarly to the $\sqsupseteq^{\text{nonatt}}$ case, every argument attacked by $E$ is also attacked by $E \cup \{a\}$, hence the same reasoning can be utilised. Thus, the addition of $a$ cannot lower the number of arguments strongly defended by $E \cup \{a\}$ from $E'$, so $E \cup \{a\} \sqsupseteq_F^{\text{cope}(\text{strdef})} E'$ if $E \sqsupseteq_F^{\text{cope}(\text{strdef})} E'$. Therefore the win/lose record of $E \cup \{a\}$ is at least as good as the win/lose record of $E$ and since $a \notin E_F^-$, $a$ is not attacked by $E$ and therefore strongly defended by $E \cup \{a\}$ from $E$, implying $E \cup \{a\} \sqsupset_F^{\text{strdef}} E$. Thus, $E \cup \{a\} \sqsupset_F^{\text{cope}(\text{strdef})} E$. □

Lemma A.78. $\text{cope}(\sqsupseteq^{\tau})$ *violates addition robustness for* $\tau \in \{\text{nonatt}, \text{strdef}\}$.

Example 66. *Consider AF* $F_{21} = (\{a, b, c, d, e, f\}, \{(c, a), (d, b)\})$ *as depicted in Figure 38 and the sets* $\{a, d, e\}$ *and* $\{a, b, e, f\}$*. These two sets are equally plausible to be accepted with respect to* $\text{cope}(\sqsupseteq^{\tau})$ *for* $\tau \in \{\text{nonatt}, \text{strdef}\}$*, i. e.,*

$$\{a, d, e\} \equiv_{F_{21}}^{\text{cope}(\sqsupseteq^{\tau})} \{a, b, e, f\}.$$

*If additional robustness were satisfied, adding the attack* $(a, b)$ *should not result in:*

$$\{a, b, e, f\} \sqsupset_{F'_{21}}^{\text{cope}(\sqsupseteq^{\tau})} \{a, d, e\}.$$

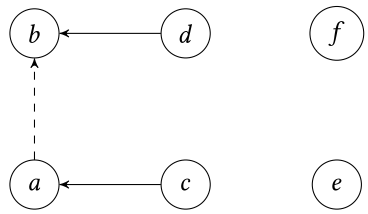


Fig. 38. AF $F_{21}$ from Example 66, where the attack $(a, b)$ is added later to obtain $F'_{21}$.

*Where* $F'_{21} = (\{a, b, c, d, e, f\}, \{(c, a), (d, b)\} \cup \{(a, b)\})$. *However, this is the case for* $F'_{21}$, *therefore* $\text{cope}(\sqsupseteq^{\tau})$ *violates additional robustness for* $\tau \in \{\text{nonatt}, \text{strdef}\}$.

Lemma A.79. $\text{cope}(\sqsupseteq^{\text{Conflicts}}, \sqsupseteq^{\text{Undef}})$ *and* $\text{cope}(\sqsupseteq^{\tau})$ *satisfy syntax independence for* $\tau \in \{\text{nonatt}, \text{strdef}\}$.

Proof. Clear by definition. □

□

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